

REPORT

# Life Expectancy by Skin Color or Race in Brazil

## Regional Estimates



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## Summary

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## 1. INTRODUCTION

This third report continues the project "Life Expectancy by Skin Color or Race in Brazil: Regional and Socioeconomic Differentials and their Interactions with Health", developed in partnership between Cedeplar/UFMG and the Institute of Mobility and Social Development (IMDS). As in the first two reports, the focus remains on quantifying and interpreting mortality inequalities between whites and blacks in the country, understood as an expression of structural asymmetries in access to opportunities, resources and well-being.

In the first report, we focused the analysis on two main axes. First, we constructed annual life tables by sex and race/skin color in Brazil between 2005 and 2015, based on the linear interpolation of the  $l_x$  and  $nL_x$  functions of the inter-census tables estimated for 2000–2009 and 2010–2019. Second, we preliminarily discussed the challenges of disaggregating these estimates to the subnational levels. We show that the combination of heterogeneities in racial self-declaration, regional differences in the composition of groups (in particular in the role of the "brown, mixed race, or nonwhite" category), and the relevance of interregional migratory flows impose substantive limitations on the direct use, at the regional or state level, of the iterative inter-census method, which has proven to be appropriate for the national level. We highlight, in particular, that selective migration and variation in census coverage tend to affect estimates more intensely when territorial and racial dimensions are introduced simultaneously.

In the second report, we organized the regional estimates around five methods (M1 to M5), ranging from a more restrictive hypothesis, which assumed homogeneous racial differentials in all regions, to specifications that incorporated additional information on infant mortality by race/skin color, regional intensity indices of racial differentials, and specific calibrations by region and sex. Our objective was to explore ways of decomposing regional total mortality, imposing, in each case, a set of numerical restrictions on regional life expectancy at birth, vertical consistency with national standards and adherence to official IBGE mortality levels.

The objective of this third and final report is to extend the application of Methods 1 to 5, developed in the previous stage only for the major regions, to the units of the federation, always recognizing the numerous methodological difficulties already pointed out since the first report and reiterated in our research meetings. In addition, we present a sensitivity analysis of the results of Method 5, adopted since the second report as the



most consistent set of estimates with the constraints and validations established in the project. As previously emphasized, the estimates produced here should be interpreted as approximations, anchored in explicit hypotheses and equally explicit validation procedures. These are not measures observed in the strict sense, but rather decompositions of total mortality, informed by national patterns of racial inequality and regional indicators of infant mortality. The purpose is to offer, for each UF and sex, a set of life tables by race/skin color that is at the same time comparable between units, compatible with official statistics, and transparent as to the methodological choices involved.

## 2. DATABASE

### **Cedeplar/IMDS tables by sex and race/skin color**

Thus, as in the second report, the starting point is the set of life tables prepared by the Cedeplar/UFMG team for the Institute of Mobility and Social Development (IMDS). We used tables for Brazil in 2015, disaggregated by sex (men, women and both sexes) and by race/skin color (whites and blacks/browns). The estimates contain probabilities of death in five-year age groups ( $q_x$ ), number of survivors ( $l_x$ ), deaths ( $d_x$ ), person-years ( $L_x$ ), and life expectancy based on each age ( $e_x$ ).

### **Atlas of Human Development in Brazil (PNUD–IPEA–FJP)**

The second source is the Atlas of Human Development in Brazil, developed by a partnership between the United Nations Development Program (PNUD), the Institute for Applied Economic Research (IPEA) and the João Pinheiro Foundation (FJP). The central objective of the Atlas is to offer a comprehensive set of socioeconomic and demographic indicators at different geographic levels, with a focus on measuring human development from a subnational perspective. The estimates are the result of demographic harmonization and modeling work that combines, mainly, information from PNUDs, IBGE Demographic Censuses and vital statistics systems, with corrections for underreporting and inconsistencies, as documented by the authors of the Atlas.

For our application, the module is of specific interest that provides estimates of infant mortality by race/skin color, by Unit of the Federation, and for Brazil as a whole. To this end, the authors use indirect methods, based on questions about the survival of children born alive in household surveys. For the year 2015, the information is based on PNUD data. By using the Atlas/PNUD as a reference, we explicitly anchor our estimates in a



synthesis that already incorporates the joint methodological effort of PNUD, IPEA, and FJP.

### **2015 Population by State, Sex, Race/Skin Color, and Age**

The third base is the distribution of the resident population by Unit of the Federation (UF), sex, race/skin color (white and black/brown), and five-year age groups from 0–4 to 80 years of age and older. From this base, we derived three main structures: i) the state population by sex, race/skin color, and age, used to weight specific mortality by race/skin color and to obtain aggregate tables "all races" in each state; ii) the 0–4 year old child population per state, whereby it is necessary to use the Atlas/PNUD infant mortality estimates as a reference for the level at young ages; and iii) regional and national aggregations (obtained from the UFs), used as consistency checks, and as a basis for vertical validations with national references. At all stages, these populations function as weights that translate, in demographic terms, the racial composition observed in each territorial unit.

As in the regional stage, although the ideal would be to use the racial composition of live births, we chose not to incorporate data from DATASUS, given the possibility of introducing new biases. Thus, we used the distribution observed in the age group 0 to 4 years as a proxy for our estimates at early ages.

### **IBGE Regional Life Tables (2015)**

Finally, in Method 5, we used life tables produced by IBGE for 2015 by Unit of the Federation (UF) and sex, which provide, for each UF and sex, life expectancy at birth and mortality profiles by age. These tables do not directly enter into the decomposition by race/skin color but serve as an external reference to control the final level of aggregate mortality in each UF (all races), ensuring compatibility with official statistics. In practical terms, we compared life expectancy resulting from our models by UF and sex with IBGE values, defined targets within previously established tolerances and, based on this, adjusted level parameters to ensure greater consistency with the mortality officially estimated for each UF.

With these databases, we now have simultaneously a national mortality standard by age, sex, and race/skin color (Cedeplar/IMDS), subnational references for the level of mortality at early ages (Atlas/UNDP), and information on population composition by state, sex, race/skin color, and age for aggregation and weighting. The following section describes how these inputs are combined in the five methods (M1 to M5), now systematically applied at the level of UFs, preserving the same general logic of the



regional stage and explaining, when necessary, operational adjustments and additional validations required by the greater degree of territorial disaggregation.

### 3. MAIN METHODS AND RESULTS

#### 3.1. Method 1: Cedeplar/IMDS fixed with weighting by racial composition

Method 1 is the starting point of the estimation strategy. It deliberately assumes that the spatial differences in mortality result only from the composition by race/skin color and sex, keeping mortality profiles estimated by Cedeplar/IMDS for 2015 fixed throughout the territory. In other words, the tables by race/skin color and sex are national; what varies between UFs (and, by aggregation, between major regions) is exclusively the composition of these groups in the population.

In this method, no external information on the level of mortality (such as Atlas/UNDP or IBGE) is used: the entire level by race/skin color and sex is implicit in national tables Cedeplar/IMDS. Thus, for each geographic unit  $g$  (UF or region), we replicated IMDS death probability vector by age, sex, and race/skin color. In practical terms, for all age  $x$ , sex  $s$  and race  $r$ , we have  $q_{x,g}^{(r,s)} = q_{x,IMDS}^{(r,s)}$ . From this vector and using demographic parameters derived from IMDS tables themselves, in particular, the size of the age range  $n_x$ , the average contribution time of the dead  $a_\omega$ , and the condition of the open group (via  $e_\omega$ ), we reconstructed, for each combination  $g \times r \times s$ , the complete life table  $(q_x, l_x, d_x, L_x, T_x, e_x)$ , ensuring that, by construction, the race/skin color-specific subnational tables exactly reproduce the national IMDS standard. The same procedure is applied to both sexes. That is, we directly replicate the odds  $q_x$  of IMDS for both sexes, by race/skin color, for each UF and for each region, reconstructing the respective complete tables with corresponding parameters.

The next step introduces the territorial element of the method, but only through racial composition. To obtain tables for all races in each state (and, analogously, in each region), we combined the specific mortality rates of IMDS tables by race/skin color, weighting them by the population distribution observed in 2015 by age, sex, and race/skin color. For each unit, sex, and age, we define the aggregate rate as a weighted average:

$$m_{x,g}^{(tot,s)} = \frac{\sum_r m_{x,IMDS}^{(r,s)} p_{x,g}^{(r,s)}}{\sum_r p_{x,g}^{(r,s)}}.$$



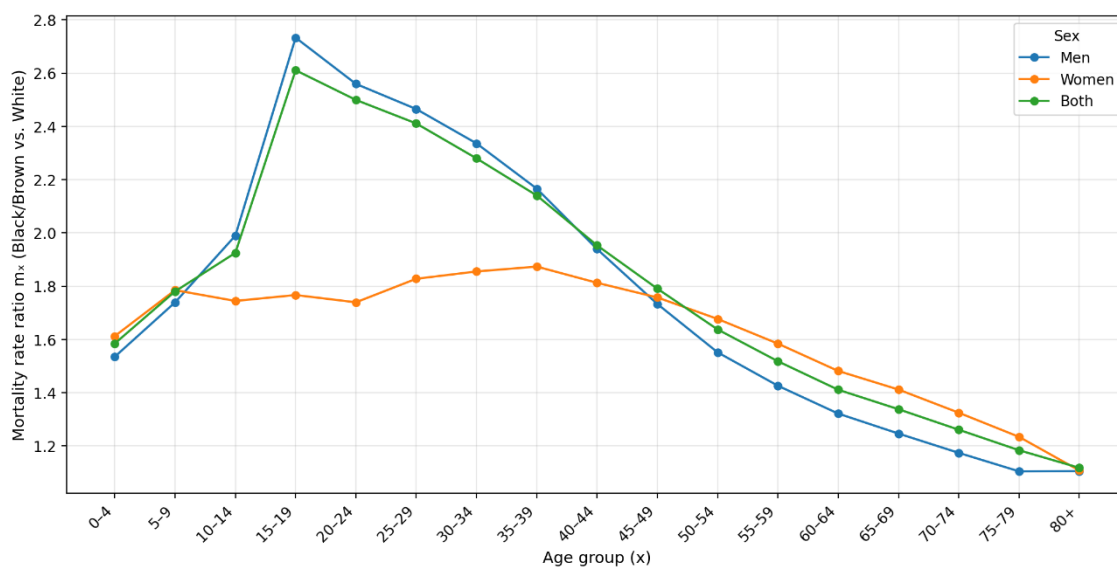
In the operational procedure, in addition to  $m_x$ , we calculated by population weighting the parameters necessary to reconstruct the aggregate table (in particular,  $a_x$  and the condition of the open group via  $e_\omega$ ), and then we reconstructed the table for all races from vector  $m_{x,g}^{(tot,S)}$ . This same reasoning is applied to the tables for all races and both sexes.

As a product of Method 1, we generated a complete set of tables by UF and by region, covering: (i) sex race/skin color; (ii) both sexes, race/skin color; (iii) sex all races; in addition to (iv) both sexes all races.

### Main patterns observed (UFs and regions) from Method 1

Figure 1 shows the ratio of mortality rates between black/brown and white population (Black + Brown/White), by age group and sex, derived from national tables Cedeplar/IMDS. These functions were widely discussed in the previous stage of this project. Within the scope of Method 1, this profile is fixed and national: the curve does not vary between regions or between UFs, as the method does not incorporate any territorial information on the level or age pattern of mortality. It only replicates, for each geographic unit, the national tables by sex and race/skin color.

**Figure 1.** Mortality Rate Ratios by race/skin color and age, Cedeplar/IMDS



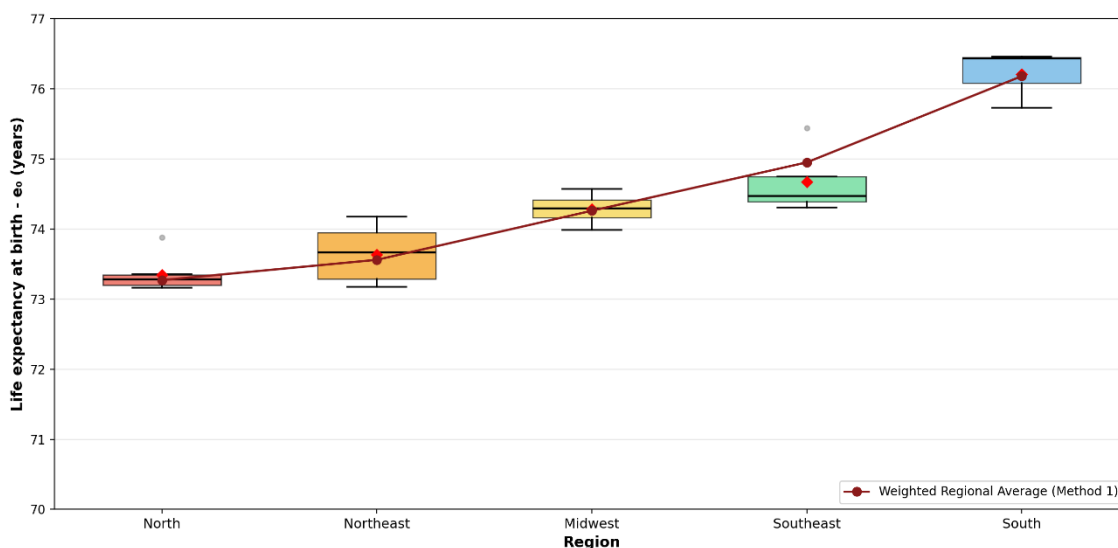
The figure shows that black/brown mortality is systematically higher than white mortality (ratio  $m_x > 1$ ) throughout the life cycle, with a well-defined age pattern. Among men, the relative excess of black/brown mortality is more pronounced in young and adult ages, and declines in advanced ages; Among women, the differential is smaller and varies more



smoothly, converging to values close to 1 at older ages. Consistent with this result, the racial differential in  $e_0$  is also national and constant in Method 1: for men, the gap (white – black/brown) is 5.87 years; for women, 4.05 years; and for both sexes, 5.36 years.

Figure 2 summarizes the distribution of  $e_0$  among the Units of the Federation within each major region, for both sexes and all races, also highlighting the weighted regional average. The graph highlights a central aspect of Method 1: by keeping the national mortality profiles by race/skin color fixed, the variation of  $e_0$  between UFs within the same region is relatively small and results exclusively from differences in the racial and sexual composition of the state populations. In descriptive terms, the intra-regional amplitude of  $e_0$  is 0.72 years in the North, 1.00 in the Northeast, 0.58 in the Central-West, 1.14 in the Southeast and 0.73 in the South. The points marked as outliers correspond, in this method, to UFs whose population composition implies an  $e_0$  for all races above (or below) the typical standard of the region. In the case illustrated, Rondônia ( $e_0 = 73,88$ ) stands out as an outlier in the North, and São Paulo (below line  $e_0$  and equal to 75.44) in the Southeast. These deviations should be strictly read as compositional effects within the regions, and not as evidence of statewide mortality patterns.

**Figure 2.** Variation of  $e_0$  among the Units of the Federation in each region. Method 1.



Taken together, Method 1 establishes a counterfactual reference scenario, a pure compositional effect, which answers the question: what would be the levels of mortality by region and by state if the only territorial differences were due to the composition by race/skin color and sex, while keeping the national mortality profiles by race/skin color fixed? The results indicate that racial composition explains an important part of the differences between major regions: the weighted regional average of  $e_0$  in the South is 76.18 years, compared to 73.27 in the North, a difference of 2.91 years. At the same time,



intraregional dispersion remains limited in this scenario, suggesting that empirically observed differences between UFs, when variation in the level of mortality is allowed, also tend to reflect real heterogeneities in age patterns and mortality levels, dimensions that will be progressively incorporated in Methods 2 to 5.

### 3.2. Method 2: Regional adjustment by race in infant mortality from UNDP/Atlas

Method 2 combines three data sources. First, the national Cedeplar/IMDS life tables for 2015, disaggregated by sex and race/skin color, which provide the age pattern of mortality and the demographic parameters necessary for the reconstruction of the tables by five-year age groups. Secondly, the Atlas/UNDP estimates of infant mortality, by race/skin color and geographic unit, expressed as probability of dying before completing 1 year of life,  $q$  lower than line 1, calculated for both sexes. Thirdly, the 2015 population by geographic unit, sex, race/skin color and five-year age groups, used as a weighting basis in aggregations and in the construction of total tables.

The objective is to obtain tables by race/skin color and sex in each geographic unit, preserving the age pattern implicit in the Cedeplar/IMDS tables, but adjusting the mortality level to reflect the geographic differentials observed in the Atlas/UNDP. In substantive terms, the method introduces territorial heterogeneity only at the level of mortality at young ages (anchored in  $q_1$ ), propagating this adjustment to all ages through a relational model, without changing the original age format of Cedeplar/IMDS.

The Units of the Federation are grouped into five major regions: North, Northeast, Southeast, South and Midwest. For each region  $r$  and race/skin color  $c$ , regional infant mortality is calculated as the weighted average of state estimates  $q_1^{(u,c)}$ , using as weight the population aged 0 to 4 years of their own race/skin color in the UF (adding men and women). Defining  $N_{u,0-4}^{(c)}$  as the 0 to 4 year population of race  $c$  in the UF  $u$ , we have:

$$q_1^{(r,c)} = \frac{\sum_{u \in r} q_1^{(u,c)} N_{u,0-4}^{(c)}}{\sum_{u \in r} N_{u,0-4}^{(c)}}$$

This weighting produces a regional indicator consistent with the racial distribution of the child population in the region. Although the ideal would be to use the racial composition of live births, we chose not to use data from DATASUS, given the risk of introducing additional biases. Thus, we used the distribution observed in the age group of 0 to 4 years as a proxy for risk exposure at very young ages.



The next step establishes a connection between the Atlas/UNDP indicator (death up to 1 year) and the indicator available directly in the Cedeplar/IMDS tables (death up to 5 years). On the side of the Atlas/UNDP, one considers  $q_1^{(BR,c)}$  for Brazil by race/skin color. On the Cedeplar/IMDS side, one uses  $q_5^{(Imds,c)}$ , understood here as the probability of death in the interval 0 to 4 (that is,  $5q_0$ ) for both sexes and race  $c$ . A proportionality factor by race/skin color is then defined:  $R_c = \frac{q_5^{(Imds,c)}}{q_1^{(BR,c)}}$ .

It is assumed, as a first-order approximation, that this ratio also applies outside the national aggregate. Thus, for each geographic unit  $g$  (region  $r$  or UF  $u$ ) and race  $c$ , the target value for mortality up to 5 years is defined as:

$$q_5^{(g,c,target)} = R_c \cdot q_1^{(g,c)},$$

where  $q_1^{(g,c)}$  is the infant mortality observed in the Atlas/PNUD for geographic unit  $g$  (or, in the regional case, the weighted average defined above).

To ensure that this adjustment is applied to all ages, we use a relational model on the logito scale. Thus, let  $q_5^{(IMDS,c)}$  be the probability of dying up to 5 years of the IMDS standard for race  $c$ , we have defined, for each region  $r$  and race  $c$ , a level parameter given by

$$\alpha_{r,c} = \text{logito}(q_5^{(r,c,target)}) - \text{logito}(q_5^{(Imds,c)}),$$

where  $\text{logito}(p) = \ln\left(\frac{p}{1-p}\right)$ . In intuitive terms,  $\alpha_{r,c}$  indicates how much it is necessary to adjust, on the logito scale, the IMDS mortality curve so that the resultant  $q_5$  coincides with the desired geographic value, preserving the age pattern of the mortality function.

In this way, the differential by sex implicit in the IMDS tables is preserved, while the geographic level of mortality, by race/skin color, is calibrated in a manner consistent with the information from the Atlas/UNDP for both sexes. In the open group, the convention of the adopted table system is maintained, preserving the corresponding parameter of the IMDS standard.



From the new probability vector  $q_x^{(g,s,c)}$ , the complete life table is reconstructed for each combination of geographic unit, sex and race/skin color,  $(q_x, l_x, d_x, L_x, T_x, e_x)$ , maintaining the same interval structure, by geographic unit and sex, by means of weighted averages for all races by geographic unit and sex by means of weighted average in  $m_x$ , using the corresponding populations of 2015 as weights. For each geographic unit  $g$ , sex  $s$  and age  $x$ :

$$m_x^{(g,s)} = \frac{\sum_c m_x^{(g,s,c)} N_{g,s,c,x}}{\sum_c N_{g,s,c,x}}.$$

The analogous procedure is applied to obtain results for both sexes, matching men and women based on their respective populations. The resulting total tables serve as a basis for comparison with the official references and as input for subsequent methods.

### Main patterns observed (UFs and regions) from Method 2

Figures 3 and 4 directly compare the results of Methods 1 and 2 for life expectancy at birth ( $e_0$ ) estimated for both sexes and all races, evidencing the progressive nature of methodological development. As observed, in Method 1, the age pattern and the level of mortality by race/skin color and sex are entirely anchored in the national Cedeplar/IMDS tables. Thus, territorial variation  $e_0$  emerges only from the composition by race/skin color (and, secondarily, by sex), since the mortality profiles by age are, by construction, common to the entire territory. The racial differential implicit in the national tables is, in this scenario, constant in space: for both sexes, the  $e_0$  gap (White – Black) is 5.36 years; by sex, it is 4.05 years among women and 5.87 years among men.

In Method 2, the IMDS age standard is maintained, but the level of mortality is now calibrated territorially based on the Atlas/UNDP infant mortality estimates. Figure 3 shows that calibration for infant mortality significantly alters the scale of heterogeneity between UFs. While in Method 1 the national scope of  $e_0$  between UFs is 3.30 years (from 73.16 in Pará to 76.46 in Santa Catarina), in Method 2 it increases to 11.50 years (from 67.33 in Amapá to 78.83 in Santa Catarina), that is, approximately 3.5 times higher. The change is not homogeneous between the regions. In Method 2, intra-regional dispersion expands in the North (4.90 years), Northeast (6.96) and Midwest (5.49), remains relatively contained in the Southeast (2.06) and contracts in the South (0.31), suggesting that an important part of territorial inequality hidden in Method 1 appears precisely in regions where health conditions and institutional response capacity are more heterogeneous



among UFs. In parallel, the weighted regional averages now reflect a much more intense gradient than in Method 1: in Method 2, the regional  $e_0$  (both sexes, all races) is 78.64 in the South, 77.47 in the Southeast, 73.80 in the Midwest, 71.61 in the Northeast and 70.96 in the North, with a regional amplitude of 7.68 years.

**Figure 3.** Variation of  $e_0$  between the Federation Units in each region. Methods 1 and 2.

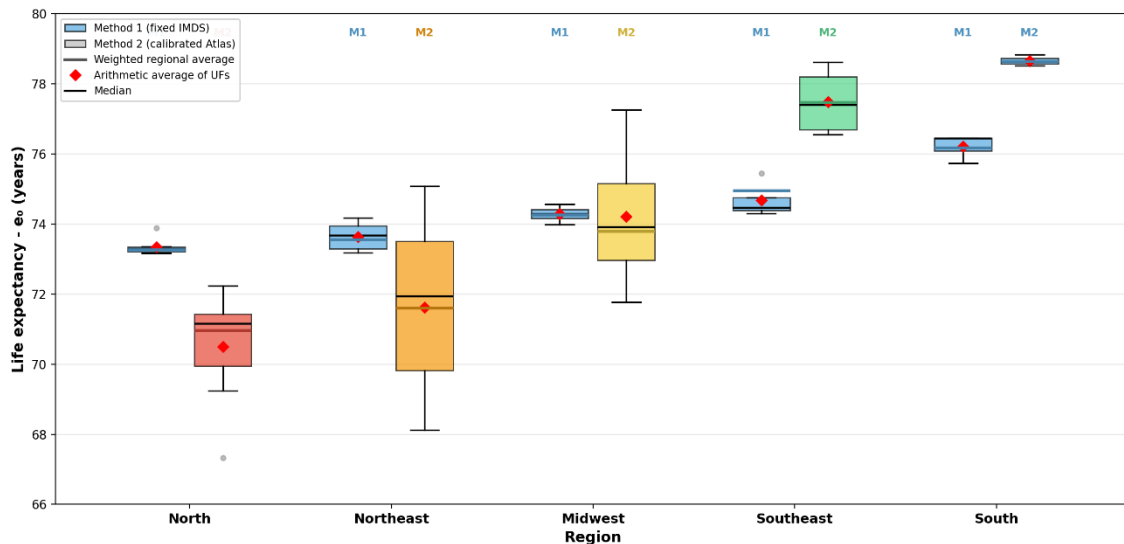


Figure 4 complements our analysis by explaining the meaning and magnitude of the displacements in life expectancy. It shows that the calibration of Method 2 imposes, for most UFs, revisions in the level of mortality implicit in Method 1. The regional pattern is clear: the North, Northeast and Midwest move, on average, to  $e_0$  levels below those of the fixed IMDS standard (interpretation compatible with  $\alpha > 0$ , that is, infant mortality worse than that suggested by the national IMDS), while the Southeast and South move to levels higher than the IMDS standard (interpretation compatible with  $\alpha < 0$ , infant mortality better than the national average. This contrast is the most direct sign that, unlike Method 1, Method 2 does not merely describe a compositional effect. It also incorporates spatial variation in the level of mortality, anchored in an external observational indicator ( $q_1$  of the Atlas/UNDP), and propagates it over the ages while preserving the age standard of reference.

This transition has immediate implications for racial inequality. In Method 1, the racial gap in  $e_0$  is constant in the territory (5.36 years for both sexes), by construction. In Method 2, as the level adjustment is done separately by race/skin color based on  $q_1$ , the racial differential is no longer invariable and starts to vary between regions. In regional terms, the gap (White - Black) is now 4.88 in the North, 5.21 in the Northeast, 4.40 in the Midwest, 3.91 in the Southeast and 3.56 in the South. This implies a reduction of approximately 33% in the South (3.56 versus 5.36) and also relevant reductions in the



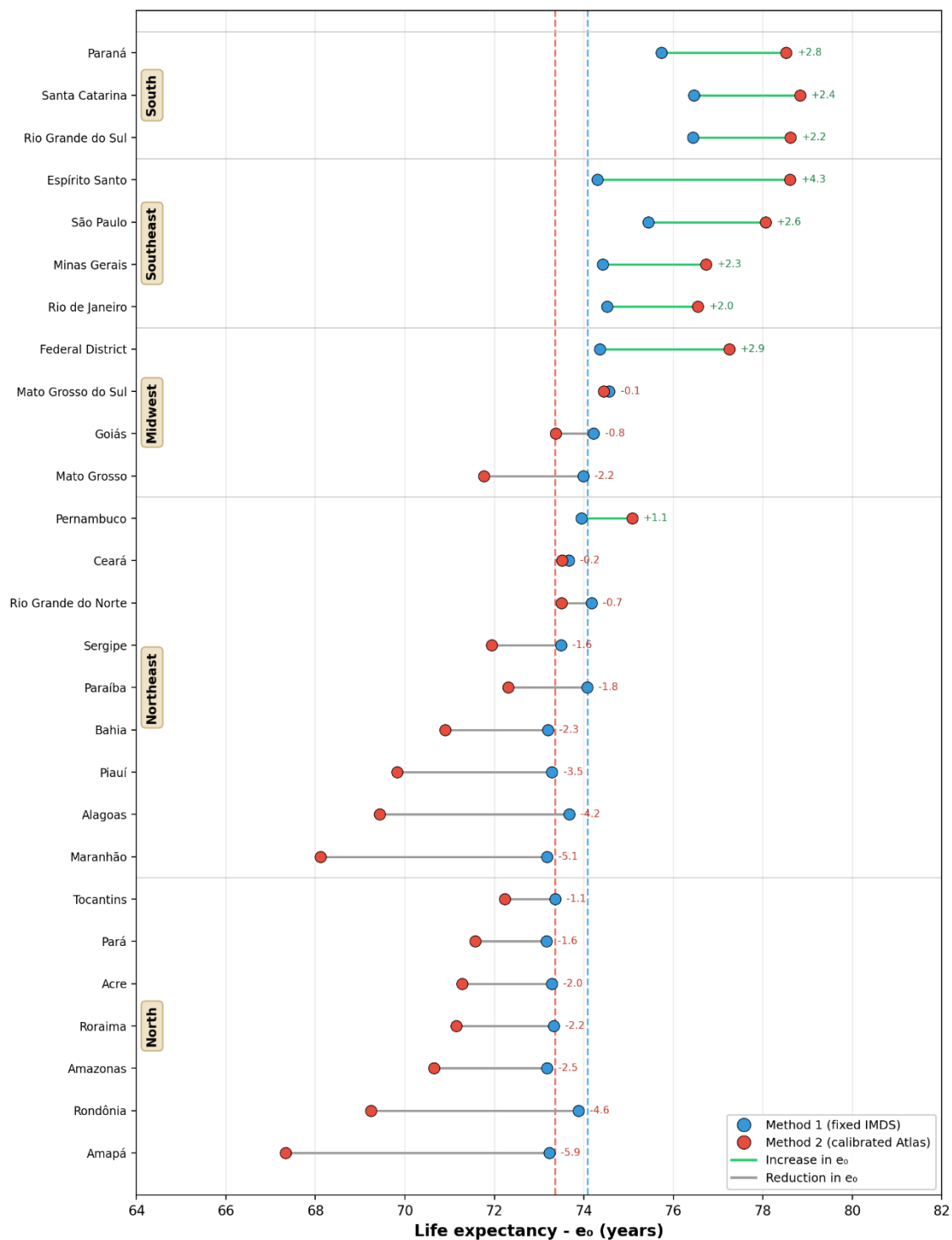
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Southeast, in contrast to the Northeast, where the differential remains close to the national pattern.



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**Figure 4.** Change of  $e_0$  by Unit of the Federation. Method 1 -> Method 2.





### 3.3. Method 3: Regional adjustment by race anchored simultaneously in the estimates of the Atlas/UNDP and IBGE

Method 3 maintains the general logic of Method 2 but introduces an additional calibration step to ensure consistency with the official IBGE estimates. In simple terms, we continue to use the Atlas/UNDP to capture the differences in child mortality by race/skin color between territories, but we now anchor the aggregate level of child mortality of each major region in the values of  $q_1$  obtained from IBGE regional tables. Thus, the Atlas/UNDP provides the structure of the differentials by race/color within each region, while the IBGE defines the level of  $q_1$  that the system must reproduce for all races in each region.

From a database perspective, Method 3 uses the same core information already described for Methods 1 and 2: (i) the national tables of Cedeplar/IMDS by sex and race/skin color, which provide the age pattern of mortality and the parameters necessary to reconstruct complete tables; (ii) the estimates of infant mortality from the Atlas/UNDP by race/skin color and Unit of the Federation (UF), expressed as  $q_1$ ; and (iii) the 2015 population by state, sex, race/skin color, and five-year age groups, used as weight in the aggregations. To these inputs is now added  $q_1$  by major region produced by the IBGE for 2015 (all races, by region), derived from the regional tables by sex. These IBGE estimates are used strictly as a reference for the aggregate level of regional infant mortality and do not interfere with the age pattern of mortality functions.

The first step of Method 3 reproduces the procedure of Method 2 to organize the Atlas/UNDP information. For each UF  $u$  and race/skin color  $c$ , we have  $q_1^{(u,c)}$ . Then for each major region  $r$ , weighted averages of these probabilities are calculated, using the child population of the UF (0 to 4 years) as weight,  $N_{u,0-4}$ , obtaining:

$$q_1^{(r,c),\text{Atlas}} = \frac{\sum_{u \in r} q_1^{(u,c)} N_{u,0-4}}{\sum_{u \in r} N_{u,0-4}}.$$

From these quantities by race/skin color, we also build, for each region, a total  $q_1$  implicit in the Atlas,  $q_1^{(r),\text{Atlas}}$ , as a race/skin color-weighted average, using regional child populations by race/skin color as weights.



The distinctive step of Method 3 is the calibration of the regional level of the Atlas to coincide with the regional level of IBGE. For each region  $r$ , a calibration factor is defined:

$$s_r = \frac{q_1^{(r),\text{IBGE}}}{q_1^{(r),\text{Atlas}}}.$$

This factor is then applied to the Atlas probabilities by race/skin color within the region, so that after correction, the race/skin color-weighted average reproduces exactly the regional  $q_1$  of IBGE:

$$q_1^{(r,c),\text{cal}} = s_r \cdot q_1^{(r,c),\text{Atlas}}$$

In the extension to UFs, the same principle is applied within each region: for each UF  $u \in r$ , one obtains  $q_1^{(u,c),\text{cal}} = s_r \cdot q_1^{(u,c)}$ . In intuitive terms, the Atlas/UNDP continues to determine the differential between whites and blacks (and their territorial variation), but the average level of infant mortality in each major region is now defined by IBGE.

Once obtained the  $q_1$  calibrated by race/skin color, Method 3 resumes the same mechanism of connection with IMDS tables already introduced in Method 2. As IMDS tables do not directly feature  $q_1$  (but they do have  $q_5$ , or  $5q_0$ ), we again use, for each race/skin color  $c$ , a national factor that relates the two indicators, and becomes  $q_1$  calibrated on a target for  $q_5$ . Then, a level parameter is estimated on the logito scale that it displaces for each territorial unit  $g$  (where  $g$  can be a large region  $r$  or a UF  $u$ ), the IMDS standard death probability by age curve. The adjustment is applied to all ages by inversion of the logito, preserving the IMDS age standard and changing only the level, so that the resulting  $q_5$  coincides with the territorial target defined from  $q_1$ . In the open-label group, the relational model already described in the previous methods is analogous.

The same reasoning is extended to sex-specific tables. Starting from  $q_x^{(\text{Imds};s,c)}$ , we apply the same  $\alpha_{g,c}$  calculated based on  $q_1$  of both sexes, preserving the sex differential implicit in IMDS, adjusting the territorial level of mortality for each race/skin color. At the end, the tables are aggregated by race/color to obtain total tables (all races), by sex, and for both sexes, by means of weighted averages of age-specific rates, using the 2015 territorial populations as weights, as in Methods 1 and 2.

From a substantive point of view, Method 3 can be interpreted as an improvement on Method 2 in terms of regional vertical consistency. We continue to explore the Atlas/UNDP to capture racial inequalities in infant mortality, while preserving the



Cedeplar/IMDS age pattern. However, by imposing that the total resultant  $q_1$  in each major region coincide with the  $q_1$  from regional IBGE tables, we ensure that our territorial tables by race/skin color and sex are compatible with the level of infant mortality officially reported for the total population in each region. In short, if Method 2 anchors the level in  $q_1$  from the Atlas/UNDP, Method 3 adds a second anchor to the IBGE, resulting in a system that coherently combines racial differentials (Atlas/UNDP) and aggregated regional levels (IBGE).

### **3.4 Method 4: Vertical adjustment Brazil x IMDS from regional tables by race/skin color**

Method 4 starts directly from the territorial tables by race/color produced in Method 3 and introduces a second level of anchoring, now at the national level. The central idea is to ensure that, by aggregating the territorial tables by race/skin color and age, weighted by the population distribution of 2015, life expectancy at birth in Brazil for whites and blacks reproduces exactly the values of the national tables of Cedeplar/IMDS. In other words, while Method 3 ensures regional consistency with the IBGE at level  $q_1$  (all races), Method 4 adds an additional vertical constraint: the tables by race/skin color, when regrouped from the territorial level for Brazil, must be compatible with the national level by race/skin color of IMDS.

From the point of view of databases, Method 4 does not introduce new external sources compared to previous methods. We continue to use: (i) the national tables of Cedeplar/IMDS by sex and race/skin color, which provide the age standard and demographic parameters by age; (ii) the distribution of the 2015 population by state, sex, race/skin color and age, previously organized to allow aggregations; and (iii) the territorial tables by race/skin color resulting from Method 3, which already incorporate simultaneously the racial differentials of the Atlas/UNDP and the regional calibration of the aggregate level via IBGE. The novelty lies in the way we use these tables to reconstruct, from the bottom up, a national table by race/skin color, guaranteeing that their life expectancy at birth coincides with the IMDS reference.

Operationally, for each race/skin color  $c$  (white and black), we define level parameter  $\partial_c$ , applied on the logito scale to the probabilities of death by age in the territorial tables by race/skin color produced in Method 3. The age pattern and territorial differentials already estimated are maintained; only the overall level of mortality by race/skin color vary, shifted by constant value  $\partial_c$  at all ages (with open-label treatment consistent with the relational model). For a given value of  $\partial_c$ , territorial tables are recalculated by race/skin color and territorial mortality is aggregated by age using the 2015 population as weights,



thus obtaining an implicit national table for race/skin color  $c$ . In substantive terms, this procedure ensures that the set of territorial tables by race/skin color remains consistent with the national levels of IMDS when aggregated to Brazil.

Once  $\partial_{\text{white}}\partial_{\text{black/brown}}$  are estimated, the same vertical adjustment applies to the sex-specific tables. The sex differential implicit in IMDS within each race/skin color is kept intact, with only the level of mortality moving uniformly by race/skin color. Next, the tables are aggregated by race/skin color to obtain total territorial tables (all races) by sex and, finally, for both sexes, by means of weighted averages of the specific rates, using the territorial populations by age, sex and race/skin color of 2015, as described in Methods 1 to 3.

From a substantive point of view, Method 4 can be interpreted as a further refinement of the vertical consistency of the table system. The Atlas/UNDP continues to define the differences in infant mortality between whites and blacks in the territories. IBGE remains as reference for the level of infant mortality aggregated by major regions, and IMDS also starts to act as an explicit anchor for the national level by race/skin color. By allowing a small uniform adjustment of the level of mortality by race/skin color in all territories, preserving the territorial and sex differentials constructed in Method 3, Method 4 produces a set of territorial tables by race/skin color and sex that is simultaneously consistent with the age structure and national levels of IMDS, and compatible with territorial differentials of infant mortality observed in external sources.

### Main patterns observed (UFs and regions) from Method 4

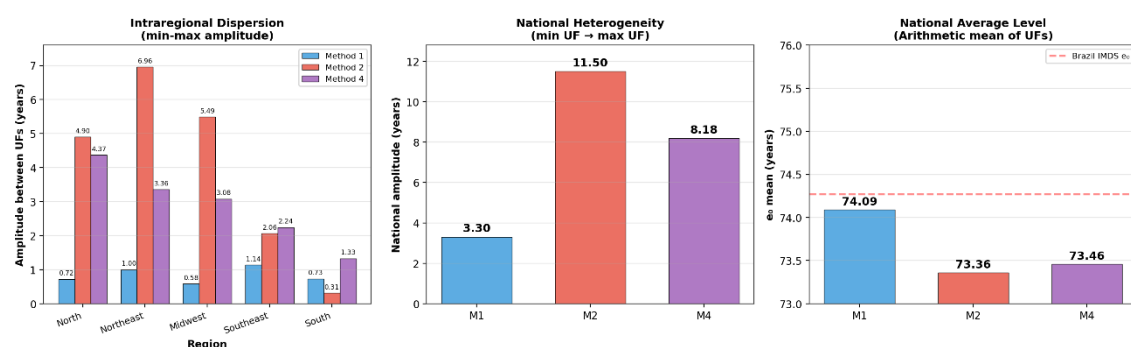
Figures 5 to 7 summarize the evolution of the results when we directly compare Methods 1, 2 and 4 for  $e_0$  (both sexes, all races) and for the racial differential in  $e_0$  (White - Black, both sexes). As discussed in the methodological sections, the three methods share the same age reference mortality pattern but differ in the anchors used to define the level of mortality in the territory. Method 1 constitutes a base scenario in which the level and age pattern by race/color and sex are entirely anchored in the national Cedeplar/IMDS tables, so that territorial variation results only from the compositional effect. Method 2 maintains the IMDS age standard but calibrates the level of mortality territorially based on the  $q_1$  of the Atlas/UNDP, separately by race/skin color, increasing spatial heterogeneity. Method 4, applied cumulatively to Method 3, adds an additional vertical constraint: part of the regional tables by race/skin color already calibrated to reproduce in each region, the total  $q_1$  of IBGE, and adjusts, by race/skin color, the level of mortality so that the national reaggregation (weighted by population) exactly reproduces the national life expectancies of Cedeplar/IMDS for whites and blacks. This way, regional



differentials and the age pattern built in the previous stages are preserved, while ensuring the coherence of the system at the level of Brazil by race/skin color.

The first evidence of this progression appears in the intra-regional dispersion of  $e_0$  between UFs (Figure 5, left panel). In Method 1, the min-max amplitude of  $e_0$  within each major region is low, ranging from 0.58 years (Midwest) to 1.14 years (Southeast), which reflects the strictly compositional nature of the method. In Method 2, this dispersion expands strongly, especially in regions where health conditions and institutional response capacity are more heterogeneous among UFs: North (4.90 years), Northeast (6.96) and Midwest (5.49). At the same time, dispersion remains relatively contained in the Southeast (2.06) and contracts in the South (0.31), suggesting that, by incorporating an external indicator of level (infant mortality), Method 2 reveals territorial inequalities that were hidden under the homogenizing scenario of Method 1. In Method 4, intra-regional dispersion is reduced in relation to Method 2 in the North (4.37), Northeast (3.36) and Midwest (3.08), signaling the disciplining effect of the national vertical adjustment. However, it remains well above the Method 1 standard in these regions. In contrast, in the Southeast and South, Method 4 results in slightly higher dispersions than in Method 2 (2.24 in the Southeast and 1.33 in the South), a pattern compatible with the fact that, in Method 2, the South is already extremely homogeneous and, in Method 4, vertical adjustment can redistribute the levels in a non-uniform way when observed in the aggregate by UF.

**Figure 5.** Comparison of  $e_0$  between the federation units and regions according to Methods 1, 2 and 4.



This same reading becomes even clearer when we observe the national heterogeneity of  $e_0$  between UFs (Figure 5, center panel). In Method 1, the national amplitude is 3.30 years, while in Method 2, it jumps to 11.50 years, that is, more than three times greater. In Method 4, the national amplitude drops to 8.18 years, which confirms that the vertical adjustment recovers part of the coherence of the system in relation to the national anchor by race/skin color, but without eliminating the territorial heterogeneity introduced by the use of the Atlas/UNDP as a level reference. In substantive terms, this

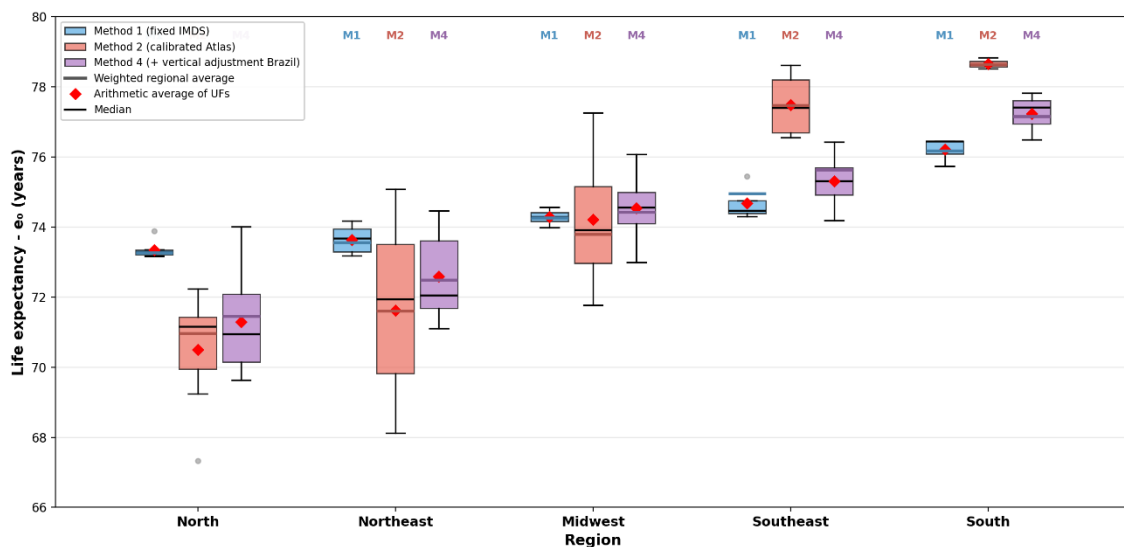


indicates that an important portion of the spatial differences suggested by Method 2 are not merely compositional: they result from territorial variations in the level of mortality consistent with an external indicator of infant mortality, but which needs to be compatible with national IMDS restrictions in Method 4.

The right panel complements this synthesis by comparing the national mean level of  $e_0$  when we take the arithmetic mean of UFs. In this criterion, Method 1 produces mean  $e_0$  of 74.09 years, while Method 2 reduces this value to 73.36 and Method 4 raises it slightly to 73.46. This difference between methods is consistent with the fact that, in Method 2, the calibration by the Atlas/UNDP substantially lowers the level of mortality implied for a wide range of UFs, especially in the North and Northeast, while Method 4 recovers part of this downgrade by imposing national vertical consistency with IMDS by race/skin color. It is important to note that this panel summarizes unweighted mean by population. Therefore, it should be interpreted as a descriptive statistic of the distribution between UFs, and not as a direct estimate of the national weighted  $e_0$ .

Figure 6 visually reinforces the same findings through regional boxplots, by simultaneously showing the level displacements and the changes in dispersion between the three methods. In particular, it shows that the transition from Method 2 to Method 4 reduces dispersion in regions with great heterogeneity (especially in the Northeast and Midwest) but preserves a pattern of spatial inequality considerably more intense than that observed in the base scenario of Method 1.

**Figure 6.** Variation of  $e_0$  between Units of the Federation in each region. Methods 1, 2 and 4.

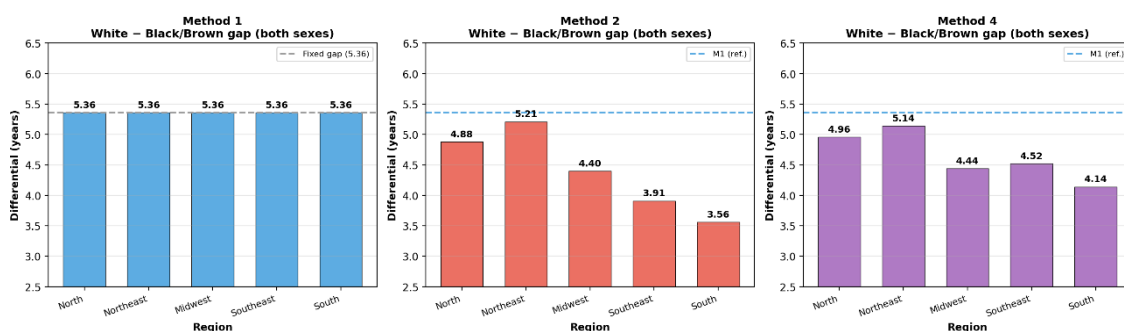


Finally, Figure 7 depicts the evolution of the racial differential in  $e_0$  (White – Black, both sexes) throughout the methods. In Method 1, the differential is constant by construction,



set at 5.36 years in all regions. In Method 2, as the level adjustment is done separately by race/skin color based on the  $q_1$  of the Atlas/UNDP, the gap is no longer invariable and starts to vary regionally, assuming 4.88 years in the North, 5.21 in the Northeast, 4.40 in the Midwest, 3.91 in the Southeast and 3.56 in the South. In Method 4, the regional racial differential shifts again, reflecting the vertical restriction that imposes national consistency by race/skin color with IMDS. The gaps become 4.96 years in the North, 5.14 in the Northeast, 4.44 in the Midwest, 4.52 in the Southeast and 4.14 in the South. Compared to Method 2, there is a re-composition of the racial differential, especially in the Southeast and South, where the gap increases and partially approaches the national standard, although it remains below the fixed value of Method 1. In interpretative terms, this is compatible with the objective of Method 4: to preserve the spatial information of the Atlas/UNDP and regional coherence with the IBGE anchor (in the previous stage), but simultaneously ensuring that the decomposition by race/skin color, when regrouped nationally, remains anchored at the national levels in accordance with IMDS by race/skin color.

**Figure 7.** Racial gap in  $e_0$  according to methods and regions.



### 3.5. Method 5: Second-order adjustment by UF and sex in accordance with IBGE, with percentage tolerance

Method 5 starts directly from the tables estimated in Method 4, which, in turn, already incorporate, cumulatively, the two previous levels of anchoring: (i) the racial differentials of infant mortality of the Atlas/UNDP, conditioned to the regional level of IBGE in Method 3; and (ii) the national vertical restriction by race/skin color in accordance with IMDS in Method 4. The objective of Method 5 is to introduce an additional, second-order adjustment, now differentiated by unit of the federation and sex, to bring the territorial levels of life expectancy at birth closer to the official IBGE references, without changing the age pattern of mortality functions or un-anchoring the system from the restrictions already imposed by race/skin color.



From the point of view of the data, Method 5 does not add new sources to those already described. It maintains the national Cedeplar/IMDS tables by sex and race/skin color, the Atlas/UNDP estimates by race/skin color and UF (already used to construct racial and regional differentials in Methods 2 and 3), the 2015 population by state, sex, race/skin color and age (used as weight in the aggregations), and the  $e_0$  references of IBGE by state and sex for 2015, which will now operate as a target for approximation to the territorial level by sex.

In the previous version of Method 5, presented in Report 2, this approximation to the IBGE was operationalized through fixed tolerance bands (in years) defined by major region, applied to the difference between  $e_{0,r,s}^{(4)}$  e  $e_{0,r,s}^{IBGE}$ . The intuition was to explicitly recognize that the uncertainty and heterogeneity of mortality data vary regionally, adopting stricter tolerances in regions with better quality of information (South and Southeast) and broader tolerances in regions where age errors and coverage problems tend to be more relevant (Midwest, Northeast, and North). However, when we extend the procedure to the level of Units of the Federation, this fixed regional parameterization proved to be less adequate to control, in a comparable way between units, the degree of horizontal adherence to IBGE, and the trade-off with vertical restrictions already incorporated into the system.

Therefore, instead of fixed bands by region, tolerances are now defined, in this report, as a fraction of the official IBGE  $e_0$ , for each UF  $u$  and sex  $s$ . Let  $e_{0,u,s}^{IBGE}$  be life expectancy at birth in accordance with IBGE and let  $e_{0,u,s}^{(4)}$  be total life expectancy (all races) produced in Method 4 for the same UF and sex, defining common percentual tolerance  $p$ , and the allowable band in years is given by

$$\tau_{u,s} = p \cdot e_{0,u,s}^{IBGE}.$$

From there, intermediate target  $e_{0,u,s}^*$  is defined by a truncation rule: if the level of Method 4 is already within the band around the official value, no further correction is applied; otherwise, the target is shifted to the edge of the band closest to Method 4 value. Formally,

$$e_{0,u,s}^* = \begin{cases} e_{0,u,s}^{(4)}, & \text{se } |e_{0,u,s}^{(4)} - e_{0,u,s}^{IBGE}| \leq \tau_{u,s}, \\ e_{0,u,s}^{IBGE} + \tau_{u,s} \cdot \text{sign}(e_{0,u,s}^{(4)} - e_{0,u,s}^{IBGE}), & \text{se } |e_{0,u,s}^{(4)} - e_{0,u,s}^{IBGE}| > \tau_{u,s}. \end{cases}$$

This construction has two important implications. First, while  $e_{0,u,s}^{(4)}$  is already compatible with IBGE within the chosen degree of tolerance, Method 5 is inert: it fully preserves the level of Method 4. Second, when there is greater discrepancy, adjustment brings the



IBGE result closer, but explicitly imposes a limit on correction, avoiding excessive rescheduling in contexts of greater uncertainty and territorial heterogeneity.

Once  $e_{0,u,s}^*$  is fixed, adjustment is implemented by uniform displacement on the logito scale, applied to the probabilities of death by age in Method 4 tables, separately for each UFU–sex combination, and in a manner common to the races within this combination. We denote in Method 4 by  $q_{x,u,s,c}^{(4)}$  the probability of death by age  $x$ , UF  $u$ , sex  $se$ , race/skin color  $c$ , thus defining

$$\text{logito}(q_{x,u,s,c}^{(5)}) = \text{logito}(q_{x,u,s,c}^{(4)}) + \gamma_{u,s},$$

for all ages below the open-label group, with treatment of the latter age group consistent with the previous methods. For each UF and sex, parameter  $\gamma_{u,s}$  is numerically determined so that, after reconstructing the tables by race/skin color and aggregating them for all races by weighted mixture of rates by age, the resulting life expectancy at birth coincides with the defined target:

$$e_{0,u,s}^{(5)}(\gamma_{u,s}) = e_{0,u,s}^*.$$

Method 5 is therefore a two-way second-order adjustment. First, it operates on a system that has already been calibrated to reproduce, simultaneously, the racial differentials of infant mortality from the Atlas/UNDP conditioned at the regional level of IBGE (Method 3) and the national vertical consistency by race/skin color in accordance with IMDS (Method 4). Second, the adjustment is restricted to a homogeneous displacement by UF and sex, preserving the age pattern and previously constructed racial differentials, while correcting only the total territorial level by sex within a controlled band.

Finally, the choice of parameter  $p$  is treated as sensitivity analysis: the procedure can be repeated for different values of  $p$ , of borderline case  $p = 0$  (greater punctual approximation to IBGE, to the extent that the target coincides with the official value) to more permissive values (wider bands), allowing mapping the trade-off between adherence to official references and preservation of restrictions accumulated throughout Methods 3 and 4.

To facilitate understanding the methods developed in this research, Table A1, attached, summarizes the hypotheses, the types of adjustment and the restrictions imposed on each of them.

## Method 5: Results



Method 5 introduces a second-order adjustment by Unit of the Federation (UF) and by sex, with the objective of bringing the estimated territorial levels closer to official IBGE references, without losing the vertical consistency ensured by Method 4. As discussed, the adjustment criterion is defined by a tolerance band proportional to the official level of life expectancy at birth from the IBGE.

Table 1 summarizes the main indicators of the trade-off between horizontal adherence (territorial IBGE) and vertical consistency (national IMDS) in the scenarios analyzed. The results are linked to the tolerance values in relation to life expectancy at birth estimated by the IBGE, ranging from 0.0% (null tolerance) to 3.0% (more permissive tolerance).

**Table 1.** Trade-off indicators by tolerance scenario (Method 5).

| <i>p</i> (%) | N adjustments | $\gamma$   medium | $\Delta e0$<br>black/brown vs<br>IMDS (years) | RMSE $\log(mx)$<br>(black/brown) |
|--------------|---------------|-------------------|-----------------------------------------------|----------------------------------|
| 0,0          | 81 (100%)     | 0.193             | +1.45                                         | 0.126                            |
| 0,5          | 70 (86%)      | 0.157             | +1.26                                         | 0.110                            |
| 1,0          | 60 (74%)      | 0.126             | +1.06                                         | 0.091                            |
| 1,5          | 50 (62%)      | 0.100             | +0.87                                         | 0.076                            |
| 2,0          | 42 (52%)      | 0.078             | +0.72                                         | 0.062                            |
| 2,5          | 34 (42%)      | 0.062             | +0.60                                         | 0.052                            |
| 3,0          | 31 (38%)      | 0.048             | +0.48                                         | 0.041                            |

Scenario  $p = 0,0\%$  represents maximum adherence to IBGE, at the cost of a vertical deviation in life expectancy at birth of +1.45 years for the black/brown population (aggregate) in relation to national IMDS, and adjustments in all 81 combinations of UFs and sex (men, women and both sexes). In contrast, values of  $p$  between 2.5% and 3.0% produce a more favorable balance: reduce vertical deviation by 59% to 67%, keep  $\log(mx)$  RMSE low (0.041 to 0.052), apply adjustments only where strictly necessary (31 to 34 UF $\times$ sex combinations), and preserve 58% to 62% of combinations unchanged ( $M5 = M4$ ).

The geographic pattern of adjustments reinforces coherence of the percentage tolerance criterion. In parameterization  $p = 2.5\%$ , all 7 UFs of the North and 6 of the 9 UFs of the Northeast receive some adjustment, as does one UF in the South (Rio Grande do Sul, men); the Southeast and Midwest regions remain unchanged. This behavior indicates that Method 4, anchored in national IMDS, already produced territorial levels close to



that of IBGE in regions with better data quality, while the North and Northeast concentrated larger residual discrepancies. Therefore, for the results presented below,  $p = 2.5\%$  is adopted. In practical terms, the choice is very close to  $p = 3.0\%$  but preserves an additional degree of adherence to official IBGE tables for UFs.

Table 2 compares Method 5 ( $p = 2.5\%$ ) to the previous methods regarding the territorial heterogeneity of life expectancy at birth ( $e_0$ ), considering both sexes and all races. The amplitude of 11.50 years of Method 2 reflects the incorporation, without additional mediation, of territorial differentials of infant mortality from the Atlas/UNDP. Method 4 (8.18 years) moderates these extremes by imposing vertical consistency with national IMDS by race/skin color. Method 5 reduces the amplitude by 47% in relation to M2 and by 25% in relation to M4, standing at an intermediate level: 1.9 times the purely compositional variation of M1, but substantially lower than the heterogeneity observed when applying the Atlas/UNDP without additional vertical constraints.

**Table 2.** National range of life expectancy ( $e_0$ ) by method (both sexes, all races)

| Method           | National amplitude (years) | Min UF ( $e_0$ ) | Max UF ( $e_0$ )       |
|------------------|----------------------------|------------------|------------------------|
| M1               | 3.30                       | Pará (73.16)     | Santa Catarina (76.46) |
| M2               | 11.50                      | Amapá (67.33)    | Santa Catarina (78.83) |
| M4               | 8.18                       | Roraima (69.63)  | Santa Catarina (77.82) |
| M5 ( $p=2.5\%$ ) | 6.14                       | Alagoas (71.67)  | Santa Catarina (77.81) |

Intraregional dispersion, measured by the amplitude between FUs in the same large region, follows a pattern similar to that described above (Table 3). In the North and Northeast, regions in which Method 5 applied adjustments in a higher proportion of UFs, the dispersion retreated to 1.67 and 2.78 years, values 66% and 60% lower than in Method 2. Still, it remains above the compositional baseline of M1. In contrast, the Southeast and Midwest maintain dispersions identical to Method 4, confirming that the percentage tolerance band fully preserves the results in these regions. In the South, dispersion is low in all methods and the M5/M2 ratio is high, as the denominator (M2) is particularly small.

**Table 3.** Intraregional dispersion (amplitude between UFs, in years) by method.

| Region    | M1   | M2   | M4   | M5   | M5/M2 |
|-----------|------|------|------|------|-------|
| North     | 0.72 | 4.90 | 4.37 | 1.67 | 0.34× |
| Northeast | 1.00 | 6.96 | 3.36 | 2.78 | 0.40× |

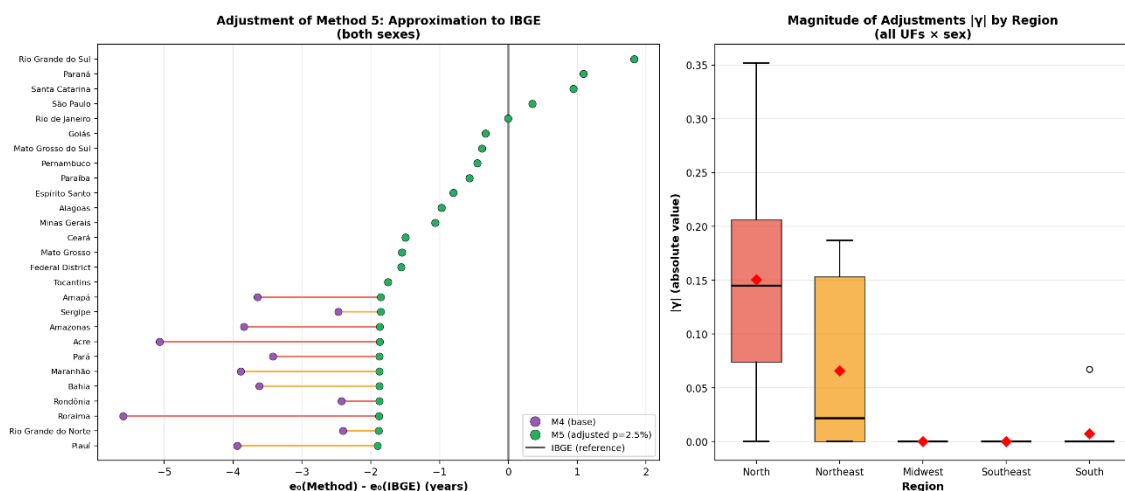


|           |      |      |      |      |       |
|-----------|------|------|------|------|-------|
| Midwest   | 0.58 | 5.49 | 3.08 | 3.08 | 0.56× |
| Southeast | 1.14 | 2.06 | 2.24 | 2.24 | 1.09× |
| South     | 0.73 | 0.31 | 1.33 | 1.34 | 4.29× |

Figure 8 details the behavior of the adjustment from M4 to M5 in relation to IBGE references by UF, evidencing truncation logic at the edge of the tolerance band. In the panel for both sexes, the estimates per UF are within or exactly on the border of 2.5%. The seven UFs that numerically reach the border (Amapá, Pará, Roraima, Bahia, Maranhão, Piauí and Sergipe) have negligible excess (from 0.000 to 0.005 years), compatible with numerical rounding.

The greatest absolute approximations to IBGE occur precisely where Method 4 presented the largest negative deviations: Roraima (from -5.60 to -1.88 years), Acre (from -5.07 to -1.87), Piauí (from -3.94 to -1.90) and Maranhão (from -3.89 to -1.87). In contrast, the UFs of the South show moderate positive deviations after adjustment (+0.95 to +1.83 years), indicating that the table system in Method 5 maintains levels slightly higher than those of IBGE in these locations. This result is consistent with the trade-off built into the choice of  $p$ : the algorithm prioritizes correcting larger residual discrepancies (concentrated in the North and Northeast) without imposing additional compression where Method 4 was already adherent.

**Figure 8.** Method 5 adjustment indicators ( $= 2.5\%$ ) $p$



By allowing independent adjustments by UF $\times$ sex, Method 5 introduces flexibility absent in Method 4, which operated uniform displacements by race/skin color. The national average of the difference between M5 and IBGE is -0.87 years for men and -1.38 years for women (of both races), with -1.16 years in the aggregate of both sexes. On average, therefore, women's estimates remain lower than men's. This result is compatible with the interaction between (i) differences in level between IMDS tables and those implied in territorial IBGE by sex, (ii) the way in which the Atlas/UNDP captures territorial



mortality differentials by sex, and (iii) the possibility of each UF×sex combination reaching its own tolerance edge without cross-restrictions. Some UFs exhibit more pronounced discrepancies between male and female behavior in adherence to IBGE. In Ceará ( $\Delta = +1.61$ ), Espírito Santo (+1.32) and Paraíba (+1.23), the difference between M5 and IBGE is systematically greater among men than among women. This pattern does not indicate internal inconsistency, but the ability of Method 5 to accommodate sex-specific territorial variation, which previous methods, by construction, did not incorporate.

Racial gaps (white population minus Black/Brown population) in Method 5 vary between 3.38 years (Pernambuco) and 8.32 years (Roraima), for both sexes. Table 4 summarizes the regional distribution of these differentials. The North has higher average gaps and greater intraregional variability, with Roraima in an extreme position. This result may reflect both limitations of input data for a UF with reduced population and more pronounced socioeconomic inequalities between racial groups. In contrast, the South, Southeast and Midwest exhibit more homogeneous gaps, converging to the range of 3.7 to 4.9 years. This pattern can be interpreted as less heterogeneity in socioeconomic conditions and access to health among racial groups in these regions or, alternatively, as lower sensitivity of territorial differentials of the Atlas/UNDP in contexts of greater coverage and better quality of records.

**Table 4.** Racial gaps (years) by region - Method 5, both sexes.

| Region    | Mean (years) | Min (years)               | Max (years)              |
|-----------|--------------|---------------------------|--------------------------|
| North     | 5.25         | 3.89 (Tocantins)          | 8.32 (Roraima)           |
| Northeast | 4.53         | 3.38 (Pernambuco)         | 6.10 (Maranhão)          |
| Midwest   | 4.09         | 3.73 (Mato Grosso do Sul) | 4.92 (Goiás)             |
| Southeast | 4.61         | 3.80 (São Paulo)          | 5.68 (Rio de Janeiro)    |
| South     | 4.16         | 3.87 (Santa Catarina)     | 4.62 (Rio Grande do Sul) |

## 4. GENERAL CONSISTENCY AND SUMMARY OF RESULTS



The methodological progression from M1 to M5 can be understood as a process of gradual incorporation of external constraints and references, aiming to reconcile territorial heterogeneity observed with vertical consistency in relation to national mortality patterns. Table 5 summarizes final regional levels in Method 5 ( $p = 2.5\%$ ) and their differences in relation to previous methods. Method 5 sets the North and Northeast regions 2.14 and 1.83 years above Method 2, bringing them closer to the compositional baseline of Method 1, while keeping the South and Southeast only slightly below the high levels of Method 2. Thus, the amplitude between large regions in Method 5 (4.05 years) is between the compression of Method 1 (2.91) and the expansion observed in Method 2 (7.68).

**Table 5.** Regional life expectancy (years) in Method 5 and differences from previous methods (both sexes, all races).

| Region    | M5 ( $p=2,5\%$ ) | M5 - M1 | M5 - M2 | M5 - M4 |
|-----------|------------------|---------|---------|---------|
| North     | 73.10            | -0.17   | +2.14   | +1.65   |
| Northeast | 73.44            | -0.12   | +1.83   | +0.95   |
| Midwest   | 74.43            | +0.17   | +0.64   | +0.01   |
| Southeast | 75.60            | +0.65   | -1.87   | -0.03   |
| South     | 77.15            | +0.97   | -1.49   | +0.00   |

Sensitivity analysis showed that  $p = 2.5\%$  constitutes a point of balance between potentially conflicting objectives: it reduces by 67% vertical deviation of aggregate black/brown population in relation to national IMDS (from +1.45 to +0.60 years), approximates the estimates to territorial IBGE in a controlled manner within an explicit truncation rule, applies parsimonious adjustments in 42% of the UF $\times$ sex combinations and preserves territorial heterogeneity at an intermediate level (national amplitude of 6, 14 years of age).

As an external reference for the scale of territorial variation by sex, in the 2015 IBGE data, the largest difference between women and men between UFs is observed in Sergipe, with 78.70 years for women and 69.77 for men (+8.93 years), followed by Ceará (+8.38), Bahia (+8.28), Rio Grande do Norte (+8.27) and Alagoas (+8.26). Considering the 54 UF $\times$ sex combinations, the highest level of  $e_0$  in IBGE occurs in the Federal District (women: 80.77) and the lowest in Alagoas (men: 68.52), resulting in a range of 12.26 years. This interval is useful as an order of magnitude of the territorial contrast observed in official statistics when full variation by UF and sex is allowed.



In Method 5 ( $p = 2.5\%$ ), considering all races aggregated and, again, the 54 UF $\times$ sex combinations, the extremes remain practically at the same level as IBGE: the highest  $e_0$  occurs in Santa Catarina (women: 80.51) and the lowest in Alagoas (men: 68.09), with an interval of 12.42 years. The proximity between amplitudes (IBGE: 12.26; M5: 12.42) is consistent with the objective of Method 5: to bring estimates closer to IBGE by UF in a controlled manner, without disorganizing the vertically anchored system, preserving a territorial variation by sex that follows the scale observed in official references, but with an explicit rule of truncation by band  $\tau$ .

When race/skin color is explicitly reincorporated (whites and blacks/browns separated), the range observed in Method 5 expands, as expected. Considering all the UF $\times$ sex $\times$ race combinations, the highest  $e_0$  occurs in Roraima, white women (81.57) and the lowest in Alagoas, black/brown men (66.69), resulting in a range of 14.88 years. As Roraima appears as the upper extreme and can influence the amplitude reading, it is also useful to report the scenario without Roraima. Excluding this state, the highest life expectancy is in Santa Catarina, white women (80.94), while the minimum remains in Alagoas, black/brown men (66.69), with a range of 14.25 years. The reduction is modest ( $-0.63$  year), indicating that the expansion of the range does not depend exclusively on the extreme case of Roraima. It reflects, above all, that in addition to territorial heterogeneity by sex, the system incorporates racial inequalities that vary substantially between UFs, without violating the horizontal criterion, since the adjustment was calibrated to maintain the UF $\times$ sex combinations within the band in accordance with IBGE.

Figure 9 presents a comparison organized by  $e_0$  of the black/brown population (growing). It is particularly useful for visualizing, in a single panel, three central dimensions of the final result: (i) the position of the M5 aggregate (both races) in relation to IBGE, reference point by UF $\times$ sex; (ii) the decomposition of this aggregate into the levels of  $e_0$  of whites and blacks/browns; and (iii) the size of the racial gap as a horizontal segment. By ordering the UFs from lowest to highest black/brown  $e_0$ , the graph shows that the combinations at the worst extreme (lower lines) simultaneously concentrate low longevity in the black/brown population and more extensive racial gaps in some UFs, while the best extreme (upper lines) combines higher black/brown  $e_0$  and, in general, more contained gaps. The figure also makes it transparent where IBGE/vertical consistency trade-off is more intense, that is, in cases where the IBGE point moves away from the M5 aggregate and the system responds by means of truncation within  $\tau$ , maintaining parsimonious and territorially concentrated adjustment.

To better assess the robustness of the racial component in the UF $\times$ sex $\times$ race stage, we performed a sensitivity check based on two complementary criteria. First, we built an



indicator of effective size from the Atlas/UNDP in the cut-out by race, approximating births by  $Pop(0-4)/5$  and calculating the expected number of infant deaths by race  $E[D_0] \approx (Pop(0-4)/5) \times q_1^{Atlas}$ . This indicator is conservative and serves as a warning for situations in which the Atlas estimate, as it is based on the PNAD (sample), may present greater variance when stratified by race/skin color in specific UFs. Next, we combined this indicator with a substantive criterion: we verified whether the racial gap of Method 5 is above the average of the region itself, which would signal a result that deviates from the regional pattern in a potentially anomalous way.

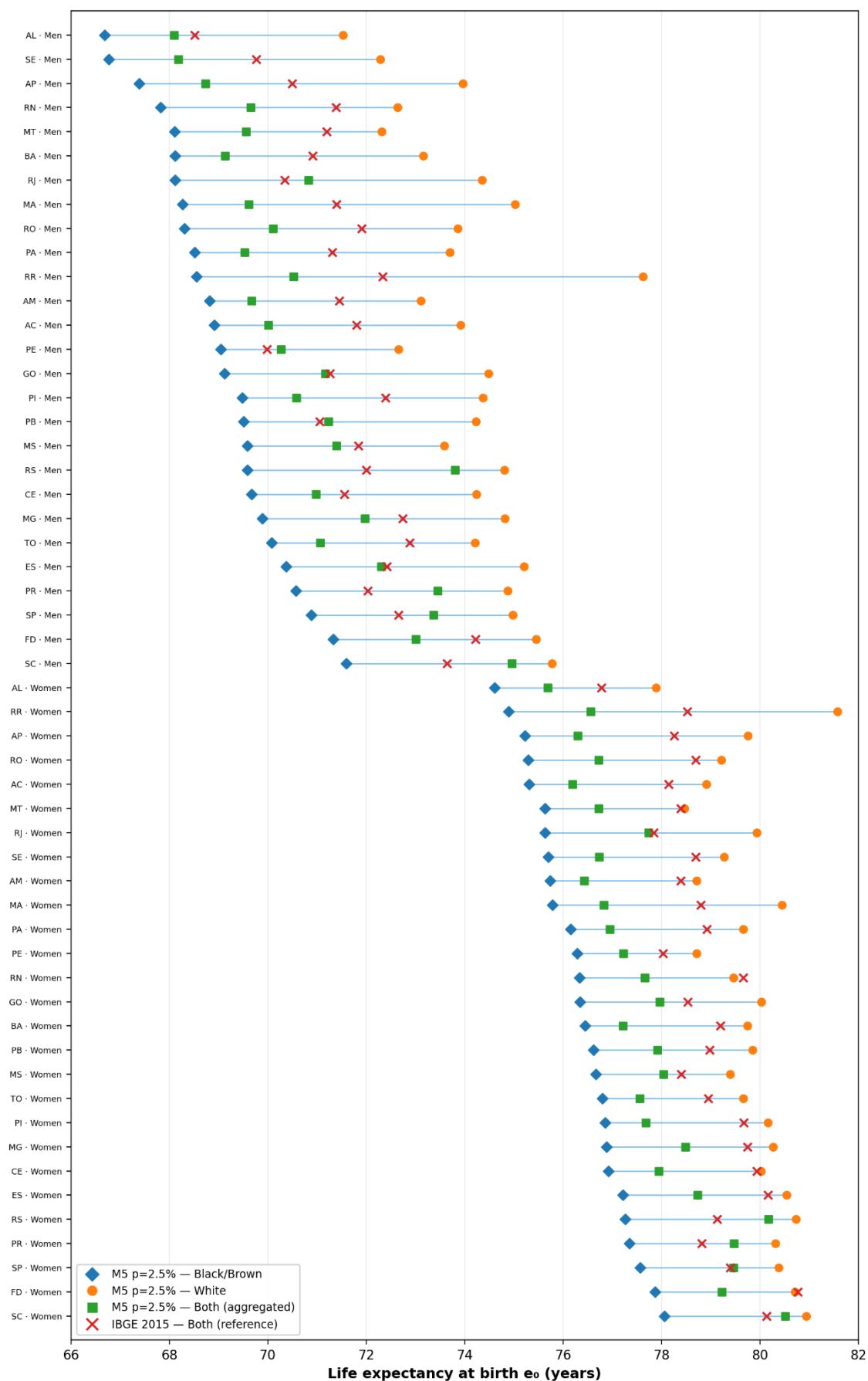
The joint application of these criteria shows that Roraima stands out as the extreme case: the racial component of the Atlas is the weakest in terms of effective information, and the resulting racial differential is the highest and most atypical in the North. For this reason, we recommend that, for Roraima, the tables by race/skin color and sex be replaced by the regional tables of the North (race/skin color and sex), preserving the coherence of the system and avoiding that a point particularly sensitive to sample noise (PNAD) influences the substantive interpretation. For the other UFs, although there are variations and some signs of less statistical support in subgroups, the results do not constitute discrepancies that are notable when compared to regional standards, so that systematic substitution by regional means is not justified.



**Figure 9.** Comparison  $e_0$  by UF/sex (IBGE vs Method 5;  $p = 2.5\%$ ), organized by black/brown  $e_0$  (growing)



## Life Expectancy by Skin Color or Race in Brazil: Regional Estimates





## 5. CONCLUSION

Throughout the three reports, the project advanced from an initially circumscribed objective, to produce consistent estimates of mortality by sex and race/skin color, to a much more ambitious methodological agenda: to build, for 2015, an integrated system of tables by UF capable of reconciling different sources and external constraints, without losing internal consistency. This challenge is particularly important because, in Brazil, there is no consolidated and stable official statistics on mortality by race/skin color at the subnational level that would allow the simple use of racial differentials observed in vital registries. In practice, even when race/skin color is included in administrative databases, its quality is unequal and territorial comparability is limited. Thus, any attempt to estimate  $e_0$  by race/skin color in UFs requires a methodology that combines indirect information and demographic coherence, rather than relying on direct counts. In this context, what was at stake was not only the production of estimates, but the creation of a mechanism capable of transforming fragmented evidence into plausible, auditable, and mutually coherent results.

Moreover, the official model itself imposes well-known difficulties in demographic literature. IBGE tables by UF and sex are the best anchor available, but they are affected by typical limitations of vital systems and censuses: errors in age declaration, under-registration and under-enumeration, especially at advanced ages. In smaller populations, these problems are amplified and can distort the pattern of the mortality curve and, consequently, the level of  $e_0$ . It is precisely by recognizing these limitations that the work sought not to limit itself to the choice of a source, but to make explicit and operationalize a system of vertical and horizontal constraints, to document the trade-off between flexibility and parsimony, and to transform a structural problem of data (fragmented, incomplete, and sometimes inconsistent) into a consistent, reproducible, and interpretable product.

Finally, the project faced a problem that is little explained in official studies: when it descends to the level of UF and, simultaneously, disaggregates by race/skin color, the risk of introducing biases by auxiliary sources increases significantly, either by sample variations (when the information derives from household surveys), or by statistical instability in small domains, or by territorial differences in racial classification itself. This last point is substantively important: the white/black + brown composition observed, especially by the inclusion of browns in the group of blacks, may vary with regional context, with local patterns of identification and with changes in self-classification, so



that part of regional differences in racial gaps may reflect, in addition to the risk of mortality, variations in the social and statistical boundary between racial categories. The central contribution was to transform these potential weaknesses into a controlled procedure. Instead of mechanically accepting the available territorial heterogeneity, the methodological design began to incorporate consistency checks and transparent rules of adjustment, capable of preserving informativeness of sources without compromising coherence with national standards and official references. The result was a methodology that did not intend to solve the absence of perfect data but made the choices explicit and offered a replicable way to estimate mortality by race/skin color in subnational units, with clear documentation of methodological limits.

### Appendix 1

**Diagram A1. Synthesis of Developed Methods**

| Method | Central hypothesis                                                                                                                     | Main Type of Adjustment                                                                                                                                            | Restrictions on regional $e_0$                                                                                                                |
|--------|----------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| M2     | Regional adjustment by race based on Atlas/UNDP: racial inequalities vary territorially according to infant mortality                  | Level adjustment by region and race/skin color (via logito), preserving IMDS age standard                                                                          | Regional $e_0$ by race/skin color now reflects Atlas/UNDP; still without explicit calibration with IBGE                                       |
| M3     | Atlas/UNDP defines racial differentials; IBGE defines regional aggregate level (all races) of infant mortality                         | Calibration of total regional $q_1$ to IBGE and preservation of racial differentials of the Atlas/UNDP; then adjustment in logito of IMDS tables                   | In each region, total $q_1$ reproduces IBGE; $e_0$ regional by race/skin color becomes compatible with this child level                       |
| M4     | National vertical restriction by race/skin color: when aggregating regions, IMDS national $e_0$ by race/skin color must be recuperated | Level adjustment (on a logito scale) with parameters by race/skin color, applied to regional tables by race/skin color of Method 3, so that the Brazil aggregation | Ensures Brazil-region coherence by race/skin color; Regional $e_0$ is rescheduled only as much as necessary to close the national restriction |



## Life Expectancy by Skin Color or Race in Brazil: Regional Estimates

|    |                                                                                                                                                                                                                                            | reproduces that of<br>IMDS by race/skin color                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                               |
|----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| M5 | The levels of $e_0$ per UF and sex should be compatible with official references of IBGE, but without imposing perfect coincidence; the adjustment should preserve the age pattern and racial differentials constructed in Methods 3 and 4 | Additional adjustment (scaled logito) with parameters $\gamma_{u,s}$ specific per UF and sex, applied to Method 4 tables (common to the races within each UF/sex), preserving the age pattern and racial differentials previously constructed | For each UF $u$ and sex $s$ , $e_{0,u,s}$ (all races) is kept within a band around IBGE given by $\tau_{u,s} = p \cdot e_{0,u,s}^{IBGE}$ ; if Method 4 is already within the band, there is no adjustment; otherwise, the result is truncated to the nearest edge of the band |