Income-Based Affirmative Action in College Admissions

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Abstract

Using student-level microdata from Brazil, we find that low-income college students outperform their higher-income peers after controlling for admission scores, which suggests that income-based affirmative action in college admissions can generate aggregate efficiency gains. We develop an overlapping-generations model and calibrate it to data from Brazil, where such a policy is widely implemented. We find that the optimal income-based affirmative action increases welfare and aggregate output. It improves the pool of admitted students but distorts pre-college educational investments. The welfare-maximizing policy benefits lower- to middle-income applicants with income-based quotas, while higher-income applicants face fiercer competition in college admissions. The optimal policy reduces intergenerational persistence of earnings by 5.7% and makes nearly 80% of households better off.

JEL: I2, E24, J62.

Keywords: Affirmative Action, Intergenerational Mobility, Educational Investment.

1 Introduction

Affirmative action in college admissions is an important and controversial policy intended to promote educational opportunities for socially disadvantaged groups. Although the goals of the policy are clear, its impacts on individual decisions may have ambiguous effects on economic performance. For instance, the ways in which affirmative action shapes individual decisions regarding educational investments are subject to much debate.[1] On the one hand, it may level the playing field and incentivize beneficiaries to make additional investments in education. On the other hand, it could lower admission standards and disincentivize investments in education. Besides affecting investment decisions about education, affirmative action directly impacts the admission process by providing the admission committee with the applicants' social background information. With these data in hand, universities can adjust their admission standards for applicants from disadvantaged backgrounds in order to admit the most qualified candidates within that social group. This adjustment could lead to efficiency gains in college admissions by selecting applicants with higher returns to education.

In this paper, we study income-based affirmative action policies in college admissions in Brazil, where they are implemented on a large scale. We start by showing empirically that low-income students outperform their higher-income peers in college when they have the same college-admission score. Our empirical findings suggests efficiency gains can be obtained if college admissions favor low-income applicants because such a policy would substitute low-income students for high-income students with similar admission scores. Then, we investigate whether the efficiency gains generated by income-based affirmative action policies are economically meaningful and outweigh unintended distortions in educational investments. To that end, we build an overlapping-generations model and calibrate it to Brazilian data to study the long-run implications of income-based affirmative action for investment decisions, efficiency, and welfare gains.

Our main result is that income-based affirmative action is an effective policy to promote educational opportunities and to improve welfare. Optimally designed income-based criteria for college admissions not only improves welfare but also increases aggregate output. Furthermore, such a policy would make nearly 80% of households better off and would reduce intergenerational persistence of earnings by 5.7%.

We enrich a standard overlapping-generations model with a detailed college admissions process; we numerically solve for the stationary equilibrium; and we calibrate the model to the data. We model college admissions akin to the Brazilian market, providing a realistic

[1]See Coate and Loury (1993), Moro and Norman (2003), and Fang and Norman (2006).

framework consistent with our data.

College admissions in Brazil is a large and competitive market, with a national exam determining applicants' admission—an ideal setting to study and model income-based affirmative action in college admissions. In addition, we would like to highlight two aspects specific to the Brazilian market. First, in Brazil, the best college education is provided by public, tuition-free colleges.[2] Second, pre-college education can be either public or private, with big differences in quality. While nearly 90% of students attend a public high school, the fraction of these students admitted to a public college is very small. For instance, in the best university in Brazil (University of São Paulo), only 23% of the admitted students come from public high schools. Although these are specific to the Brazilian market, we show in robustness exercises that our findings are not driven by these features.

Our model encompasses four distinct generations: young child, old child, young parent, and old parent. A child is born with *innate ability*, which is correlated with her parents' innate ability and interpreted as the ability inherited from the parents. Young children receive investments in early education from their parents. As a combination of educational investment decisions and innate ability, each young child develops college admission skill and human capital. The distinction between the two is key in our paper. On the one hand, *college admission skill* determines the likelihood of college admission, while, on the other hand, *human capital* determines future labor income. As a result of our calibration, educational investments have a higher marginal impact on admission skill than on human capital. Therefore, investment in early education affects the likelihood of college admission more than it affects the child's future labor earnings. To further improve future labor earnings, older children can attend college to gain a human capital boost and to access the labor market for skilled individuals.

The distinction between admission skill and human capital is the main source of inefficiency in our model. Future labor earnings and the returns to college education are based on human capital. However, even if higher-education institutions aim to admit applicants with the highest returns to college, absent specific income-based admission policies, they are constrained to make admission decisions based on a noisy measure of admission skill (for instance, any kind of standardized tests). In our model, policies targeting lower- to middle-income students can have a positive impact on aggregate output because they can make college admissions more efficient. In equilibrium, lower-income students lack the necessary resources to invest in a successful college application and are, therefore, less likely

[2] All top-10 and 18 of the top-20 universities are public in Brazil, using the ranking from the main Brazilian newspaper Folha de São Paulo. Article 206, paragraph 4, of the Brazilian Constitution states that a fundamental principle of education is that it will be tuition-free in all public schooling places.

to attend college. From an efficiency standpoint, applicants with the highest returns to education should go to college, regardless of their socioeconomic characteristics. However, high-income applicants can heavily invest in early education to build a competitive admission profile. Given the limited number of college spots, high-income applicants crowd out lower-income ones. Income-based affirmative action may tackle this source of inefficiency, as it alters college demographics by admitting more lower- and middle-income applicants with higher returns to education and fewer high-income ones with lower returns to education.

In our quantitative exercise, an income-based affirmative action policy specifies a number of college spots to be allocated to each income quintile. We numerically solve for two distinct optimal policies: one that maximizes efficiency (i.e., aggregate output) and one that maximizes welfare.

The welfare-maximizing policy significantly changes the composition of admitted students. It reduces the number of admitted students from the top quintile by 25% and increases the number of admitted students from the other income groups. Specifically, the optimal policy implies a threefold increase in the number of admitted students from the bottom quintile of the income distribution. Such a policy makes 80% of households better off. Households in the first four quintiles are, on average, willing to forgo 0.57% of their consumption in every period and in every state of the world to have the policy implemented. Households in the top income quintile, however, are worse off, on average, and would have to be compensated by 1.9% of their consumption. In the aggregate, households are better off and willing to forgo 0.07% of their consumption to have the policy implemented. Also, the optimal policy promotes educational opportunities by reducing earnings persistence by 5.7%.

The efficiency-maximizing policy is qualitatively similar to the welfare-maximizing one as it favors applicants in the bottom four quintiles. However, it reallocates fewer college spots to quotas. Accordingly, its effects on earnings persistence and households' willingness to pay are attenuated. It reduces intergenerational persistence of earnings by 2.1% and households are willing to forgo 0.02% of their consumption to have the efficiency-maximizing policy implemented.

The next subsection discusses the related literature. The rest of the paper is organized as follows. Section 2 presents empirical evidence supporting our economic mechanism. Section 3 presents our benchmark model. Section 4 shows our calibration procedure. Section 5 reports our policy evaluation results, along with robustness exercises and the analysis of the affirmative action policy being implemented in Brazil. Finally, Section 6 presents our concluding remarks.

Related Literature Our paper's main contribution is to the literature on intergenerational persistence of earnings, focusing on the effects of income-based affirmative action. Our model builds on Restuccia and Urrutia (2004). In their model, however, applicants are automatically admitted to college, while we model college admissions in detail to evaluate the economic implications of affirmative action. Restuccia and Urrutia (2004) show that about half of the observed intergenerational earnings persistence is due to investments in early education. In line with this finding, Holter (2015) and Blankenau and Youderian (2015) find that investments in early and college education also reduce persistence of earnings. Our paper contributes to this literature by showing that an optimal income-based affirmative action policy in college admissions reduces persistence of earnings.

Our paper is also closely related to the literature that analyzes incentive implications of affirmative action in college admissions.[3] Assunção and Ferman (2013) find that affirmative action disincentivizes effort for the preferentially-treated students in the short run.[4] Adding to their results, our paper shows that, in the long run, preferentially-treated students do not decrease their educational investments.

There are recent works that evaluate empirically the effects of affirmative action policies in Brazil. Vieira and Arends-Kuenning (2019) use the staggered adoption of policies across universities and find that the enrollment effects were concentrated in the most competitive programs. Estevan, Gall, and Morin (2022) study the affirmative action policy adopted by a large Brazilian university and find little evidence of behavioural reactions regarding examination preparation effort. Mello (2022) studies the interaction between the National Law of Quotas and a policy that centralized applications in a nationwide online platform. We contribute by providing a complementary quantitative framework to interpret the long-term effects of income-based affirmative action policies.

Finally, our paper is related to the literature that studies the effects of banning affirmative action in college admissions (Chan and Eyster 2003, Arcidiacono 2005, Fryer and Loury 2007, Epple, Romano, and Sieg 2008). Our contribution to these studies is to endogenize educational investment decisions in a general equilibrium setting; we find income-based affirmative action to be a sound policy to reduce persistence of earnings and improve welfare.

[3] Hickman (2010), Krishna and Robles (2012), Hickman (2013), Assunção and Ferman (2013), Krishna and Tarasov (2013), Kapor (2015) and Veloso (2016).

[4] Our paper also relates to the literature on the effects of government policies on education incentives. In addition to Assunção and Ferman (2013), Peruffo and Ferreira (2017) study the long-term effects of conditional cash transfer programs on schooling decisions in Brazil.

2 Empirical motivation

In this section, we present empirical evidence for the main mechanism in our model: the mismatch between students' expected human capital and their likelihood of college admission. We show that low-income college students outperform their higher-income peers after controlling for admission scores, classroom fixed effects, and demographic characteristics. This finding suggests potential efficiency gains from income-based affirmative action in college admissions, as these policies substitute relatively lower-income students for higher-income students with similar admission scores.

We use microdata of two eminent exams in Brazil. First, we use data from the national college admission exam, namely ENEM ("*Exame Nacional do Ensino Médio*"), which is a national standardized exam developed by the Ministry of Education and offered yearly. The ENEM exam was first introduced in 1998 with the objective of measuring the performance of high school graduates. Since 2009, ENEM has been used as a college admission exam. Currently, ENEM is used as the only admission exam for the majority of federal universities and several other higher-education institutions, including private universities. It consists of 180 multiple-choice questions in four areas—Mathematics, Humanities, Sciences and Languages—as well as a written essay. In 2015, more than 8.7 million students took the ENEM exam. We interpret ENEM scores as college admission scores.

The second exam, ENADE ("*Exame Nacional de Desempenho de Estudantes*"), is a national exam developed by the Ministry of Education to evaluate higher education programs every year in Brazil. The exam is given to students who have completed at least 80% of their degree, and contains general questions, as well as questions specifically related to the students' majors. A total of 368 thousand students took this exam in 2021. We interpret ENADE scores as college performance.

We use microdata on students taking ENADE from 2014 to 2019.[5] We rely on admission scores as a proxy for a student's likelihood of admission. Our sample is restricted to students admitted to college in 2012 or before, since the National Law of Quotas was approved in that year. Including affirmative action beneficiaries in our sample would complicate the interpretation of ENEM scores as admission probabilities.

Our regression specification is given by (1):

$$\text{ENADE}_{ic} = \sum_{k=1}^{n} \beta_k \mathbb{I}\{\text{Income category}_{ic} = k\} + \alpha \text{ENEM}_{ic} + \gamma \mathbf{X}_{ic} + \mu_c + \varepsilon_{ic}, \tag{1}$$

[5] See Appendix A for details on the dataset construction.

where i denotes an undergraduate student; c denotes classroom;[6] and ENADE_{ic} denotes the ENADE score of student i in classroom c. Income category_{ic} is a categorical variable with six levels indicating the student's family income in the year when the ENADE was taken;[7] $\mathbf{X}$ is a vector of baseline control variables (age, race, and gender); μ_c are classroom fixed effects; and ε is an error term. The parameter of interest, β_k, expresses the expected ENADE score difference between a student with family income in category k and a comparable student in the same classroom with family income in the baseline category (more than ten minimum wages).

Table 1 shows the regression coefficient estimates for three specifications. In the first column, we control for age, race and gender (base controls), but do not include classroom fixed effects and ENEM scores as a control variable. These estimates show that, after controlling for demographic characteristics, higher-income students perform better than low-income students. To interpret the magnitude of this effect, we note that college students with family incomes lower than 1.5 minimum wage (which corresponds to the 30^{th} percentile of the family earnings distribution) score, on average, nine points lower on ENADE than students whose family income is higher than ten minimum wages. This corresponds to an expected score of $9.085/14.52 = 0.64$ standard deviations lower.

In the first column, college performance is a strictly decreasing function of the family's income. One reason that high-income students obtain better scores than low-income students is that they self-select into different degrees and/or higher-education institutions, potentially with more competitive admissions and superior quality, which would translate into higher ENADE scores in the graduation year.

To control for this mechanism, we add classroom fixed effects in the second column. The estimates are now markedly closer to zero, meaning that selection into different institutions and degrees explains a large amount of the college performance inequality between low- and high-income students. Comparing students in the same classroom, those with a family income lower than 1.5 min. wages are expected to have ENADE scores 3.27 points lower than students in the top income category. Note that the estimates in this column are still an increasing function of students' income categories. Even though within-classroom comparisons are considerably fine and granular, there can still be a considerable amount of heterogeneity in admission probabilities across students in the same classroom. We now add admission scores as a control variable to account for this mechanism.

The last column shows the expected relative college performance of students with the

[6] A student's classroom is defined as the student's higher-education institution, major, ENADE year, and class cohort.

[7] We don't have information on family income in the year when the student started its undergraduate studies.

Table 1: College performance by income levels

This table shows the regression estimates of parameters β_k in (1). For each earnings category, we report the corresponding percentile interval in the family earnings distribution (according to PNAD 2015), and the n. of obs. used in the regressions. Dependent variable: ENADE score (last-year undergrad. exam). More than 10 min. wages is the base category. Base controls are age, race and gender. A student's classroom is defined as the higher-education institution where she studies, major, ENADE year, and class cohort. Mean (s.d.) of dep. var.: 46.81 (14.52). Robust standard errors are reported in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

	(1)	(2)	(3)
Less than 1.5 min. wages, $< p30$, $N = 71,210$	-9.085***	-3.274***	-0.280**
	(0.090)	(0.095)	(0.086)
From 1.5 to 3 min. wages, $[p30, p60]$, $N = 118,416$	-7.178***	-1.737***	0.364***
	(0.082)	(0.086)	(0.078)
From 3 to 4.5 min. wages, $[p60, p75]$, $N = 86,705$	-6.040***	-1.064***	0.461***
	(0.085)	(0.086)	(0.078)
From 4.5 to 6 min. wages, $[p75, p85]$, $N = 55,701$	-4.621***	-0.768***	0.478***
	(0.092)	(0.091)	(0.082)
From 6 to 10 min. wages, $[p85, p95]$, $N = 55,329$	-3.018***	-0.297***	0.418***
	(0.093)	(0.089)	(0.081)
More than 10 min. wages, $> p95$, $N = 46,310$	–	–	–
Base controls	Yes	Yes	Yes
Classroom fixed effects	No	Yes	Yes
ENEM (admission score)	No	No	Yes
N. of obs.	433,671	433,671	433,671

same demographic characteristics, who study in the same classroom, and who obtained similar admission grades, conditional on family's income category. Note that the estimates are no longer a monotonic function of income. Instead, ENADE scores now follow an inverse U-shaped curve as a function of income. Students in the bottom income category still perform significantly worse than higher-income students, but performance (relative to the top income category) becomes positive and increases as we move to higher incomes, reaching a peak in the fourth category (75 to 85 % of the income distribution).

Students with family incomes between percentiles 30 and 60 outperform students with family incomes higher than percentile 95 by 0.364 points, on average. Families who live on 1.5 to 3 min. wages per month in Brazil face highly unfavorable economic and social conditions.[8] Our results indicate that a student coming from this background—who is admitted with the same admission score to the same institution and degree of someone in the top income category—has positive unobserved characteristics that allow her to attain a higher college return.

[8]According to the World Bank, 20% of the Brazilian population lived in poverty (defined as living on less than \$5.50 per day in 2011 PPP terms) in 2016.

Table 2: College performance by income levels (students who don't work)

This table shows the regression estimates of parameters β_k in (1) for the subsample of students who don't work. For each earnings category, we report the corresponding percentile interval in the family earnings distribution (according to PNAD 2015), and the n. of obs. used in the regressions. Dependent variable: ENADE score (last-year undergrad. exam). More than 10 min. wages is the base category. Base controls are age, race and gender. A student's classroom is defined as the higher-education institution where she studies, major, ENADE year, and class cohort. Mean (s.d.) of dep. var.: 48.75 (14.80). Robust standard errors. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

	(1)	(2)	(3)
Less than 1.5 min. wages, $< p30$, $N = 39,385$	-10.418***	-2.934***	-0.146
	(0.123)	(0.140)	(0.127)
From 1.5 to 3 min. wages, $[p30, p60]$, $N = 49,374$	-7.899***	-1.555***	0.462***
	(0.116)	(0.128)	(0.116)
From 3 to 4.5 min. wages, $[p60, p75]$, $N = 32,948$	-6.455***	-1.138***	0.388**
	(0.123)	(0.130)	(0.119)
From 4.5 to 6 min. wages, $[p75, p85]$, $N = 22,770$	-4.778***	-1.004***	0.221
	(0.134)	(0.136)	(0.124)
From 6 to 10 min. wages, $[p85, p95]$, $N = 24,594$	-3.107***	-0.584***	0.125
	(0.132)	(0.131)	(0.119)
More than 10 min. wages, $> p95$, $N = 24,614$	–	–	–
Base controls	Yes	Yes	Yes
Classroom fixed effects	No	Yes	Yes
ENEM (admission score)	No	No	Yes
N. of obs.	193,685	193,685	193,685

One potential reason that students in the bottom income group perform worse is that they might face adverse conditions, such as having to work to complement family earnings, or being required to help with domestic tasks for long hours. To shed some light on this possibility, we run the same regressions restricting the sample to students who don't work.

Table 2 shows the estimates for this subsample. The first and second columns present the same patterns as in the previous table. The third column, however, shows a different pattern: the difference between the performances of students in the bottom and top income categories is statistically insignificant. Furthermore, the second income category is the one with the highest expected performance. Compared to the regression estimates in Table 1, the regressions for the sample of students who don't work are stronger in suggesting that low-income students outperform higher-income students who have similar admission likelihoods.

Overall, the empirical evidence shows the existence of a mismatch between students' expected human capital (measured by ENADE scores) and their likelihood of college admission (measured by ENEM scores). Furthermore, this mismatch is systematically related to income differences, with high-income students having lower human capital returns than lower-income students who have similar admission probabilities.

Our results are consistent with a recent literature on high-stakes exams (e.g., Ebenstein, Lavy, and Roth 2016, Arenas, Calsamiglia, and Loviglio 2020), questioning the validity of exam scores as an unbiased measure of students' human capital. In Brazil, there are firms and organizations specialized in coaching students for the national college admission exam (ENEM). For instance, these institutions offer private lessons and even large-scale courses to review past exams and go over practice tests. Also, several high school curricula have been adjusted to improve students' performance in college admission exams. This setting arguably amplifies the imbalance between scholastic knowledge and future labor market outcomes, which is what our empirical findings suggest.

In the next section, we present an overlapping generations model designed to speak to our empirical findings. The fundamental element of the model is that investments in pre-college education affect, simultaneously, the likelihood of college admission and the expected human capital. However, investments affect the first more than they do the latter. As higher-income families invest more in pre-college education, higher-income applicants have a higher chance of being admitted. However, controlling for the admission probability, lower-income applicants have higher expected human capital.

3 Model

In this section, we present and discuss our theoretical model. Later, in Sections 4 and 5, we use our model at calibrated parameters to provide a quantitative assessment of the long-run implications of income-based affirmative action policies.

Although our framework is general, some of its features are designed to represent the admissions process in Brazil. In order to let our model speak to the data, we have tailored it to the Brazilian college admissions process. In particular, the main differences between our calibrated model of college admissions and the U.S. market are: (i) our model features public college with the tuition cost being borne entirely by tax payers; (ii) public school is less efficient than private school; and (iii) the number of public college spots is fixed. In Section 5.4, we show that the substantive message of our results holds even if we relax these model assumptions, specific to the Brazilian market.

3.1 Setup

We develop a model of four overlapping generations based on Restuccia and Urrutia (2004). Each agent lives four periods, the first two as a child (young and old child) and the last two as an adult (young and old parent). Each generation is a continuum of measure one, and a family unit comprises one parent and one child. At each point in time, there are two different family units: one with a young child and a young parent, and another with an old child and an old parent. Each period in the model is interpreted as a 16- to 18-year period. There is no population growth. Agents are heterogeneous in their innate ability levels; a young child is born with an innate ability level that depends on her parent's innate ability, following an autoregressive process. Decisions are made at the family level, assuming no intra-household frictions. The family makes consumption and educational investment choices, maximizing the welfare of the family. Furthermore, households have access to a perfect credit market within-period, but no credit market across periods.

Young parent A young parent is identified by her human capital (h_y), her higher-education degree ($d_c = 1$ for college degree and 0 otherwise), and her young child's innate ability (π). A young parent chooses how much to consume (c_y), how much to invest in the pre-college education of her child (e), as well as the type of school her child attends ($s = 1$ for public and 0 for private school). The young parent maximizes utility from current consumption plus a discounted continuation value, given by the value function when the young parent becomes an old parent. For an old parent, the household is identified by her human capital (h_o), her higher-education degree (d_c), her old child's innate ability (π), and the educational choices previously made when the household was young (e and s).

Formally, the young parent's optimization problem is given by:

$$V_y\left(h_y, d_c, \pi\right) = \max_{c_y \geq 0, e \geq 0, s \in \{1,0\}} u\left(c_y\right) + \beta V_o\left(h_o, d_c, \pi, e, s\right) \tag{2}$$

subject to

$$c_y + e = (1-\tau)\left[d_c w_c + (1-d_c) w_n\right] h_y \tag{3}$$

$$h_o = \xi h_y. \tag{4}$$

The function $V_y(\cdot)$ is the value function of the young parent; $u(\cdot)$ is the utility function of the family; and β is a discounting parameter. Next, we discuss each of the constraints and respective variables in detail.

The young parent's budget constraint is given by Equation (3). The parent inelastically supplies human capital services in the labor market. There are two distinct labor markets in the economy, one for those with a college degree—i.e., skilled workers—and another for those without a higher education—i.e., unskilled workers. With a college degree ($d_c = 1$), the young parent's labor income is equal to $w_c h_y$, but without a college degree ($d_c = 0$), her labor income is given by $w_n h_y$, where w_c and w_n are wage rates determined in equilibrium. The government proportionally taxes labor income at rate τ, and the young parent decides how to allocate her after-tax labor income between consumption (c_y) and investment in her young child's early education (e). We assume no credit market between periods: all the young parents' earnings are either consumed or invested in early education. However, since each period represents 16 to 18 years, the budget constraint in equation (3) implies a perfect credit market within-period. In addition to choosing how much to invest in early education and how much to consume, a young parent also decides whether her child attends public ($s = 1$) or private school ($s = 0$).

Finally, the human capital of the young parent increases linearly when she becomes an old parent. This captures a life-cycle profile of earnings, and the human capital of a parent evolves according to Equation (4), where parameter $\xi \geq 1$ is a life-cycle parameter and h_o is the human capital of the young parent in the subsequent period.[9]

Human capital formation A key aspect of human capital formation captured in our framework is that it depends on both early educational investment and innate ability. In addition, a college degree further increases human capital.

The early educational investment (e) is transformed into an effective early educational investment ($\hat{e}$), with a technology that depends on the type of school attended (s), according to Equation (5):

$$\hat{e} = s\alpha\,(e + g) + (1 - s)e. \tag{5}$$

The parameter $\alpha \geq 0$ captures the quality difference between public and private school, while the variable $g \geq 0$ represents a government transfer that doesn't vary across public school students. According to equation (5), if the child attends public school ($s = 1$), private and public investments in education are perfect substitutes; however, the return to investments in education depend on the type of school attended. As a result of our calibration, α is less than one. Hence, if a young child goes to public school, she receives a government subsidy g, but early educational investments face lower marginal returns. In contrast, a private school ($s = 0$) has a higher marginal return to educational investments, but no government subsidy.

Our model has two types of colleges, private and public, with different qualities and costs

[9] We assume that parents perfectly observe their children's innate abilities. A different approach would be that a parent doesn't observe its child's ability and forms expectations on it based on its own innate ability. Such imperfect information aspect could make additional room for government policies to improve economic allocations if parents are not better informed than the government. Therefore, adopting this alternative modeling could reinforce our findings that affirmative action policies can generate relevant gains for society.

of enrollment. Specifically, human capital formation is described by:

$$h'_y = \begin{cases} \pi \hat{e}^{\psi} & \text{if no college} \\ \overline{p} \pi \hat{e}^{\psi} & \text{if public college} \\ \underline{p} \pi \hat{e}^{\psi} & \text{if private college,} \end{cases} \tag{6}$$

where h'_y is the human capital of the old child. The parameter $\overline{p}$ ($\underline{p}$) measures the human capital gain from attending a public (private) college, and $\psi \in [0, 1]$ represents the elasticity of human capital in adulthood with respect to investments in pre-college education. This human capital formation process is consistent with Ben-Porath (1967), Becker and Tomes (1979), and Restuccia and Urrutia (2004). Our specification for human capital formation implies that returns to college education increase with early educational investments and innate ability. Intuitively, both pre-college educational investments and innate ability help students to learn from college courses, thus allowing them to achieve a higher level of human capital after college. Finally, our calibration indicates that public colleges increase human capital more than private ones do.[10]

Old parent Similar to a young parent, an old parent chooses how to allocate her family labor income between consumption and investment in her child's college education. We specify the value function of an old parent as follows:

$$V_o(h_o, d_c, \pi, e, s) = \max \left\{ V_o^{\text{not apply}}(h_o, d_c, \pi, e, s), V_o^{\text{apply}}(h_o, d_c, \pi, e, s) \right\}, \tag{7}$$

where $V_o^{\text{not apply}}(\cdot)$ is the value of not applying, and $V_o^{\text{apply}}(\cdot)$ is the expected value of applying to college.

If a child does not apply to college, then she joins the labor market without a college degree and inelastically supplies human capital services. Thus, the value of not applying is

$$V_o^{\text{not apply}}(h_o, d_c, \pi, e, s) = \max_{c_o \geq 0} u(c_o) + \beta \mathbb{E}_{\pi'} \left[V_y \left(h'_y, d_c{}' = 0, \pi' \right) | \pi \right] \tag{8}$$

[10]All top ten Brazilian universities are public, and, among the top 20, only two are private, using the ranking from the main Brazilian newspaper Folha de São Paulo. Assunção and Ferman (2013) also make the case for high-quality higher education being offered mostly by public institutions in Brazil.

subject to

$$
\begin{aligned}
c_o &= (1-\tau)\left[\left(d_c w_c + (1-d_c) w_n\right) h_o + w_n h_y'\right] \\
\hat{e} &= s\alpha\left(e+g\right) + (1-s)e \\
h_y' &= \pi \hat{e}^{\psi},
\end{aligned}
\tag{9}
$$

where the first constraint is the budget constraint, the second is the effective education investment function, and the third specifies the human capital of the old child without a college degree according to equation (6). The budget constraint includes the after-tax income from all family members combined. The continuation value of the old parent is the expected value function of her own child as a young parent in the next period. This expectation is over the innate ability distribution of the old parent's grandchild, which follows an autoregressive process:

$$
\log(\pi') = \rho \log(\pi) + \sigma_\pi \varepsilon_\pi. \tag{10}
$$

The random variable ε_π is an i.i.d. standard normal. Parameter ρ captures the persistence of innate ability between distinct generations, and σ_π captures the noisiness of this process.

The expected value of applying to college is a combination of the value if the old child is admitted to public college, V_o^{admitted}, and the value if she is not admitted, $V_o^{\text{not admitted}}$:

$$
\begin{aligned}
V_o^{\text{apply}}\left(h_o, d_c, \pi, e, s\right) = {} & q\left(h_o, d_c, \pi, e, s\right) V_o^{\text{admitted}}\left(h_o, d_c, \pi, e, s\right) \\
& + \left[1 - q\left(h_o, d_c, \pi, e, s\right)\right] V_o^{\text{not admitted}}\left(h_o, d_c, \pi, e, s\right).
\end{aligned}
\tag{11}
$$

If a child applies to college, the outcome of her application is random. The probability of being admitted to public college is given by $q\left(h_o, d_c, \pi, e, s\right)$, and it is an equilibrium object that we discuss in depth in the next subsection, along with a description of the college admission market. An applicant's probability of college admission depends primarily on her college admission skill, which is a function of π and $\hat{e}$, and college applicants with higher admission skills are more likely to be admitted. In addition, the likelihood of college admission might also depend on the type of high school attended (s) and even on the parent's income (wh_o), according to the affirmative action implemented.

If an old child's college application is successful, she may go to either a private or a public college, which differ in costs and human capital gains. For this reason, if an old child is admitted to college, the old parent's value function is described as

$$
V_o^{\text{admitted}}\left(h_o, d_c, \pi, e, s\right) = \max_{c_o \geq 0, r \in \{0,1\}} u\left(c_o\right) + \beta \mathbb{E}_{\pi'}\left[V_y\left(h_y', {d_c}' = 1, \pi'\right)|\pi\right] \tag{12}
$$

subject to

$$
\begin{aligned}
c_o + ra\eta &= (1-\tau)\left[\left(d_c w_c + (1-d_c) w_n\right) h_o + w_c h_y' \left(1-\eta-b\right)\right] \\
\hat{e} &= s\alpha\left(e+g\right) + (1-s)e \\
h_y' &= (1-r)\overline{p}\pi\hat{e}^{\psi} + r\underline{p}\pi\hat{e}^{\psi},
\end{aligned} \tag{13}
$$

where the first constraint specifies the budget constraint, the second is the effective education investment function, and the third defines the human capital of the old child, according to equation (6). The variable c_o is the consumption of the old family, and r is the decision to attend private college (i.e., $r = 1$) or public college (i.e. $r = 0$). Parameter η represents the time spent in college; a is the tuition cost per unit of time; and b captures the time spent applying to college. Both η and b represent the opportunity costs associated with a college education since the old child can work only a fraction $1 - \eta - b$ of that period. The cost of a public college education is not factored into the budget constraint because it is fully subsidized by the government. This assumption, however, can be relaxed by assuming that the government finances only a fraction of the tuition rather than bearing the entire cost. Our quantitative and qualitative results are robust to this change.

If an old child is not admitted to a public college, then she may still attend a private college and join the skilled labor market. We assume that all college applicants are automatically admitted to a private college. According to Brazilian college data (2008 INEP report, Tables 7, 9 and 11), nearly seven times more students apply to public college than there are public college spots available. However, for a private college, the ratio of applicants to private college spots is about 1.3. Moreover, only 5% of the public college spots are unfilled, while 50% of the private college spots are unfilled. Based on these data, we assume that no minimum admission score is needed to attend a private college.[11]

Conditional on the child not being admitted to the public college, an old parent chooses between enrolling her child in a private ($k = 1$) college or not enrolling her child in any college ($k = 0$). The old parent's optimization problem if her child is not admitted to a public college is, therefore, given by

$$
V_o^{\text{not admitted}}\left(h_o, d_c, \pi, e, s\right) = \max_{c_o \geq 0, k \in \{0,1\}} u\left(c_o\right) + \beta \mathbb{E}_{\pi'}\left[V_y\left(h_y', d_c' = k, \pi'\right)|\pi\right] \tag{14}
$$

[11]These data are from a report by the National Institute for Educational Research at the Brazilian Ministry of Education (INEP/MEC 2008 report).

subject to

$$
\begin{aligned}
c_o + az\eta &= (1-\tau)\left[(d_c w_c + (1-d_c)w_n)\, h_o \right. \\
&\qquad\qquad \left. + h'_y\left((1-k)w_n\,(1-b) + kw_c\,(1-\eta-b)\right)\right] \\
\hat{e} &= s\alpha\,(e+g) + (1-s)e \\
h'_y &= (1-k)\pi\hat{e}^{\psi} + k\underline{p}\pi\hat{e}^{\psi}.
\end{aligned}
\tag{15}
$$

Although credit and savings markets are absent from the model, a single budget constraint per period implies a perfect credit market within-period. Given the interpretation of 16 to 18 years per period, an old child can use the family's income until her 32^{nd}-36^{th} birthday to repay any student loan she takes to go to college.

Admissions We model college admissions as a competitive market in which applicants compete for college spots. A young child builds college admission skill (z) by combining innate ability and effective early education investment, according to Equation (16):

$$z = \pi\hat{e}^{\gamma}, \tag{16}$$

where the parameter $\gamma \in [0,1]$ measures the elasticity of admission skill with respect to educational investments.[12] We interpret an individual's admission skill as the scholastic knowledge that a child acquires up until high school. It is a combination of a child's intelligence (innate ability) and the investments made in her education. The fundamental source of inefficiency in this model is that $\gamma \neq \psi$. In Section 3.3 we discuss this issue in detail.

As the result of an old child's application, public college observes a noisy signal of her admission skill. This signal is interpreted as her application score or her grade on the admission exam, and it is defined by

$$\log(\mathcal{P}) = \log(z) + \sigma_p\varepsilon_p, \tag{17}$$

where ε_p is an i.i.d. standard normal, and σ_p is the noisiness of admissions.

The measure of potential applicants is one, which includes all old children, while the measure of spots available at the public college is given by parameter $\mathcal{S}$. Given the number of college spots available, the public college admits applicants with the highest scores ($\mathcal{P}$) until all the spots are filled. Thus, the probability of public college admission is the probability of being among the top $\mathcal{S}$ applicants according to the admission score. By the law of large

[12]This specification is in line with Restuccia and Urrutia (2004).

numbers, there is a unique grade point cutoff, say $\bar{q}$, such that every applicant with $\mathcal{P} \geq \bar{q}$ is admitted. The cutoff $\bar{q}$ is the price that clears the admissions market.

Thus, under no affirmative action, the probability of college admission is given by:

$$q\left(h_o, d_c, \pi, e, s\right) = \text{Prob}\left(\mathcal{P} \geq \bar{q}\right) = 1 - \Phi\left(\frac{\log(\bar{q}) - \log(z)}{\sigma_p}\right), \tag{18}$$

where $\Phi\left(\cdot\right)$ is a standard normal cumulative distribution function.

Affirmative action In our model, income-based affirmative action in college admissions changes the likelihood of a successful college application by conditioning it on the applicant's income or on the type of high school she attended. If affirmative action is implemented, preferentially-treated students have a different grade point cutoff from other applicants. The cutoff grade for the preferentially-treated applicants is, of course, less than or equal to the other applicants' cutoff.

In our model, quotas are equivalent to giving bonus points to the preferentially-treated applicants. Proposition 1 in Appendix B shows that for every equilibrium with quotas, there is an equilibrium with bonus points that results in the same allocation. Conversely, for every equilibrium with bonus points, there is an equilibrium with a quota that results in the same allocation. This result allows us to study the effects of quotas on college admissions by implementing bonus points instead of quotas.[13] When the policy is implemented, preferentially-treated applicants have a bonus point added to their application score. Specifically, the admission score becomes

$$\log\left(\mathcal{P}\right) = \begin{cases} \log\left(z\right) + \sigma_p \varepsilon_p & \text{if not preferentially-treated} \\ \log\left(z\right) + \log\left(\mathcal{P}_{bonus}\right) + \sigma_p \varepsilon_p & \text{if preferentially-treated,} \end{cases} \tag{19}$$

where $\mathcal{P}_{bonus}$ are extra points given to preferentially-treated applicants, as specified by the policy. The magnitude of such bonuses defines the intensity of the policy, as more points given imply that more college spots are allocated to preferentially-treated applicants.

In this paper, we focus on income-based affirmative action, targeting low-income applicants. A policy must specify the bonus points that each subgroup of applicants—a group identified by their position on the income distribution—receives.

Production There is one representative firm that hires human capital to produce consumption goods using a production function with constant elasticity of substitution between

[13] The main reason to use bonus points is that solving for the equilibrium under bonus points is computationally less intensive than under quotas.

skilled and unskilled workers—that is, college- vs. non-college—degree workers. The firm's profit maximization problem is given by:

$$\max_{H_c \geq 0, H_n \geq 0} Y - w_c H_c - w_n H_n, \tag{20}$$

where

$$Y = f(H_c, H_n) = A \left[\theta H_n^{\rho_Y} + (1-\theta) H_c^{\rho_Y}\right]^{1/\rho_Y}. \tag{21}$$

In the firm's production function, A is a scale parameter; H_n is the human capital from unskilled workers; H_c is the human capital from skilled workers; θ is a share parameter for unskilled labor; and ρ_Y drives the elasticity of substitution between skilled and unskilled labor—specifically, $1/(1-\rho_Y)$ is the elasticity of substitution between college-degree (skilled) labor and labor without a college degree (unskilled).

3.2 Equilibrium

Let $x_y = (h_y, d_c, \pi)$ and $x_o = (h_o, d_c, \pi, e, s)$ denote the young and old households' state variables, respectively. We use the following equilibrium concept to solve the model.

Definition. *A Stationary Recursive Competitive Equilibrium is a set of value and policy functions, wages w_n and w_c, government expenditures on early education g, grade point cutoff $\bar{q}$, distributions $\mu_y(x_y)$ and $\mu_o(x_o)$ over the agents' states such that (i) agents maximize: given equilibrium prices, young and old parents, as well as the representative firm, optimize; (ii) markets clear: human capital, output and admissions markets clear; (iii) the government budget is balanced; and (iv) distribution over the state space is stationary.*

Below, we discuss in more detail the market-clearing conditions described in the equilibrium definition. We first clarify the human capital market clearing; then, the resource constraint; the government budget balance; and, finally, the college admissions market-clearing condition.

The human capital markets clear when the aggregate human capital supplied by the households equals the representative firm's demand for human capital in each labor market, skilled and unskilled.

From the firm's first-order conditions, in equilibrium, the relative wages satisfy

$$\frac{w_c}{w_n} = \frac{1-\theta}{\theta} \left(\frac{H_c}{H_n}\right)^{\rho-1}.$$

From the household supply, skilled aggregate human capital is the sum of the human capital of all skilled agents working in a given period—the old child's generation, as well as the young and the old parents' generations. Similarly, the unskilled aggregate human capital is the sum of the human capital of all unskilled workers.

Formally, let $\mu_y(x_y)$ and $\mu_o(x_o)$ be the stationary distribution over the state space. Since there is a measure one of agents in each generation, the aggregate human capital of workers with a college degree (skilled) can be written as:

$$\begin{aligned} H_c = & \int h_y^{(\text{college})} d\mu_y\left(x_y\right) + \int h_o^{(\text{college})} d\mu_o\left(x_o\right) \\ & + \left(1-\eta-b\right) \int \mathbf{1}^{\text{apply}}\left(x_o\right) \Big[q\left(x_o\right) h_y^{\prime(\text{public college})}\left(x_o\right) \\ & + \left(1-q\left(x_o\right)\right) k\left(x_o\right) h_y^{\prime(\text{private college})}\left(x_o\right) \Big] d\mu_o\left(x_o\right), \end{aligned} \tag{22}$$

where k is the decision to attend private college when public college admission is denied (Equation 14); h'_y is the old child's human capital (Equation 6); q is the probability of college admission; and $\mathbf{1}^{\text{not apply}}$ and $\mathbf{1}^{\text{apply}}$ are dummy variables indicating the decision of whether or not to apply to college (Equation 7). These are equilibrium objects and depend on their respective state variables. The integrals are over the entire state space. We find in our calibration that the public college provides a higher human capital return then the private college. Therefore, in the equation above we use the fact that students who apply and are admitted choose to attend the public college.

Analogously, the non-college-degree (unskilled) aggregate human capital is given by:

$$\begin{aligned} H_n = & \int h_y^{(\text{no college})} d\mu_y\left(x_y\right) + \int h_o^{(\text{no college})} d\mu_o\left(x_o\right) + \int \Big\{ \mathbf{1}^{\text{not apply}}\left(x_o\right) h_y^{\prime(\text{no college})}\left(x_o\right) \\ & + \mathbf{1}^{\text{apply}}\left(x_o\right) \Big[\left(1-q\left(x_o\right)\right)\left(1-k\left(x_o\right)\right) h_y^{\prime(\text{no college})}\left(x_o\right)\left(1-b\right) \Big] \Big\} d\mu_o\left(x_o\right). \end{aligned} \tag{23}$$

The resource constraint is given by

$$Y = f\left(H_c, H_n\right) = C_y + C_o + E + F + G, \tag{24}$$

where C_y and C_o are the aggregate consumption of the young and old parents; E is the aggregate expenditure on early education; F is the aggregate expenditure on college education (both public and private); and G is the aggregate government expenditure on early education. Since not all young children attend public school, we have $G = \zeta g$, where ζ is the measure of students in public school. The resource constraint guarantees that everything that is produced (Y) is either consumed ($C_y + C_o$) or invested in education ($E + F + G$).

Notice that the resource constraint is automatically satisfied, given that all agents' budget constraints hold.

The government budget being balanced states that

$$\tau Y = \tau f(H_c, H_n) = \zeta g + a\mathcal{S}\eta, \tag{25}$$

where the total amount of tax collected by the government (τY) finances both investments in early education (ζg) and subsidized public college ($a\mathcal{S}\eta$). We treat τ as a tax rate parameter to be calibrated because the government chooses g in order to keep its budget balanced. This modeling choice is without loss of generality relative to the alternative of fixing g and adjusting τ endogenously. This approach makes the calibration procedure more straightforward, given that we can directly observe the tax rate τ from the data.

Finally, the college admissions market has to clear:

$$\int q(x_o)\,\mathbf{1}^{\text{apply}}(x_o)\,d\mu_o(x_o) = \mathcal{S}, \tag{26}$$

which guarantees that the measure of admitted students equals the measure of spots available.

3.3 Inefficiencies and the affirmative action mechanism

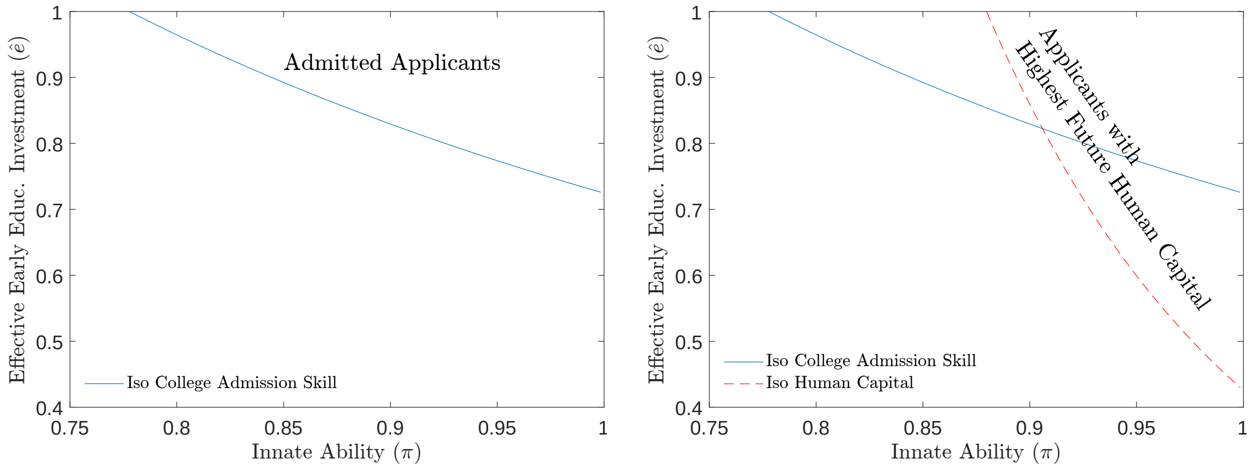
In this section, we discuss how income-based affirmative action policies might improve college admissions by making it more efficient. We highlight the fundamental inefficiency in our model and how income-based affirmative action policies can help mitigate it.

In our framework, a young parent invests in early education to increase the human capital of her child. These educational investments affect future human capital in two distinct ways. First, according to Equation (6), they increase human capital directly because human capital depends on early educational investments, regardless of whether a college degree is attained. A key parameter driving this relation is ψ, which is the elasticity of future human capital with respect to effective early educational investments. Second, according to Equation (16), these investments also increase college admission skill, at an elasticity parameterized by γ. By affecting admission skill, early educational investments indirectly boost the expected value of future human capital by increasing the likelihood of college admission.

The fundamental source of inefficiency in our model relies on the fact that parental investments in early education affect human capital less than they affect admission skill. That is, as a result of our calibration—which is described in Section 4—we have $\psi < \gamma$. Hence, investments in pre-college education are more effective at increasing the probability of college

Figure 1: Inefficiency in College Admissions

This figure reports an iso-admission skill curve in Panel A. Along this curve, the combination of effective early education investment ($\hat{e}$) and innate ability (π) leads to the same admission skill. The level of admission skill represented by the curve is such that there are exactly a measure $\mathcal{S}$ of agents above that level—for simplicity, we assume that $\hat{e}$ and π follow independent uniform distributions in the unit interval. We use the calibrated parameters from Table 3. Similarly, Panel B adds an iso-human capital curve.



(a) Admitted Applicants (b) Ideal Admitted Pool

admission than at raising the human capital gain that college provides. The second key ingredient behind the inefficiency in college admissions is that low-income parents may not have the resources to invest in their child's education. From agents' optimization problem, we have early educational investments to be a weakly increasing function of parents' income; that is, higher-income parents invest more in early education than low-income parents invest.[14] Thus, if we compare two applicants with the same admission skills but different income levels, the low-income applicant has lower early educational investments and, therefore, higher innate ability. It follows that the low-income applicant also has higher future human capital. Thus, even after controlling for admission skill, income is informative about educational investments, innate ability, and, most importantly, future human capital.

Affirmative Action Mechanism The wedge between admission skill and future human capital implies that, by admitting students based on admission skills alone, college admissions are inefficient. Future human capital is not observable, and the admissions office is forced to rely on a noisy signal of applicants' admission skills. This source of inefficiency is illustrated in Figure 1 through a stylized example. In Panel (a), the blue line is an iso-admission-skill curve for different levels of effective early educational investment and innate ability. Ap-

[14]We can derive this relation by applying the envelope theorem to the young parents' optimization problem described in Equation 2.

plicants located above the blue isocurve are those with the highest level of admission skill and represent the pool of admitted applicants under a perfectly precise admissions' signal ($\sigma_p = 0$). In contrast, for a given distribution of innate ability and effective early educational investments, the efficiency-maximizing policy maximizes aggregate human capital. In Panel (b), we add a red dashed line, which is an iso-future-human-capital curve, and the applicants located above this isocurve are those with the highest future human capital. Figure 1 highlights how the pool of admitted students is different from those with the highest future human capital. This example stresses the importance of early educational investments as an instrument to manipulate applicants' probability of admission. Income-based affirmative action can mitigate this distortion by conditioning admissions on parents' income.

4 Calibration

We numerically solve our model for the stationary recursive competitive equilibrium, as described in the Appendix D. Some parameters are calibrated directly from the data or from the literature, while others have to be calibrated by targeting related moments from the data. In our calibration, we use Brazilian data prior to the implementation of any affirmative action policy in college admissions.[15]

We assume that agents have CRRA preference, $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, with risk aversion captured by σ. We calibrate this parameter at 1.5, which is a reasonable parameter value considering several estimates from the literature (e.g., Restuccia and Urrutia 2004). The discount β is calibrated at 0.52, which is equivalent to 0.96 per year considering that each period is equivalent to 16 years. The life-cycle parameter ξ represents the increase in human capital from young to old adulthood. Using the 2004 Brazilian National Household Survey (PNAD),[16] we estimate ξ at 1.06 by using data on workers aged between 33 and 64, and regressing earnings on a dummy indicating whether the worker is between 49 and 64, controlling for race, gender, and state fixed effects.[17]

On the production technology, the scale parameter A is normalized to 10. As the production function features constant returns to scale, this is without loss of generality. In addition, we normalize $\underline{p}$ to 1, making $\overline{p}$ represent the human capital gain from public college relative

[15] Affirmative action in college admissions was first implemented in Brazil in 2004/2005 at University of Brasilia (UnB), followed by Federal University of Bahia (UFBA) and other public universities in subsequent years.

[16] PNAD is an acronym for *Pesquisa Nacional por Amostra de Domicílios.*

[17] Restuccia and Urrutia (2004) calibrate this life-cycle parameter according to their PSID estimates. Specifically, they estimate the average earnings of a 49-64-year-old person in the labor force who has a college degree, relative to those of a 33-48-year-old person with the same qualifications. They use the 1990 PSID and estimate this life-cycle parameter at 1.12, which is similar to our estimates for Brazil.

to private college. We calibrate the parameter ρ_Y in order to match the elasticity of substitution between skilled and unskilled labor, estimated by Havranek, Irsova, Laslopova, and Zeynalova (2020) at 0.46; that is, $\frac{1}{1-\rho_Y} = 0.46$, which implies $\rho_Y = 1 - \frac{1}{0.46} \approx -1.174$.[18]

In the model, each period represents 16 years, and we assume that the time spent in college is four years ($\eta = 4/16$) and that the time spent applying to college is one year ($b = 1/16$).[19] To calibrate the measure of public college spots ($\mathcal{S}$), we use estimates from the Brazilian National Household Survey data. In 2004, 17% of adults between the ages of 18 and 28 were enrolled in college or had already earned a higher degree. This fraction does not vary much with the age group considered,[20] which means that, in the model, a measure 0.17 of young children should go to either a public or a private college in equilibrium. To calibrate the measure of public college spots, we need the fraction of college students attending public institutions. In the Brazilian Educational Census, 28% of total college enrollment was in public institutions of higher education in 2004.[21] Thus, the measure of public spots is calibrated at $\mathcal{S} = 0.048$, which is 28% of 0.17.

Finally, the tax rate, τ, is directly calibrated from the data. It represents the government expenditures on education (primary, secondary and tertiary) over GDP, which was 4% in 2004, according to the World Bank. Next, we discuss the remaining parameters, which are calibrated by simultaneously matching moments from the data.

4.1 Calibration targets

The remaining nine parameters are calibrated to simultaneously match the following nine moments from the data: (i) standard deviation of log earnings; (ii) intergenerational correlation of earnings; (iii) fraction of applicants; (iv) aggregate college expenditures relative to GDP; (v) early education expenditures relative to GDP; (vi) college wage premium; (vii) fraction of students in private colleges; (viii) fraction of low-income students in public colleges; and (ix) fraction of students in public schools. Table 3 reports all of these moments in the model and in the data, as well as their related parameters.

[18] Havranek, Irsova, Laslopova, and Zeynalova (2020) estimate the elasticity of substitution using a meta-analysis of different studies and controls for publication and attenuation biases. We use their estimate for developing countries.

[19] Our main results remain qualitatively unchanged if we recalibrate the model assuming that the time spent applying to college is half a year, $b = 0.5/16$.

[20] We considered ages between 18 and 24/26/28/30/32, and the respective fractions were between 16.3% and 17.3%.

[21] Data statistics were taken from the INEP (2008) report. In 2004, there were 4,163,733 students enrolled in institutions of higher education, 1,178,328 of whom were in public institutions.

Standard deviation of log earnings. In the model, the parameter σ_π affects innate ability and earnings dispersion and, hence, is chosen to match the standard deviation of log earnings. Using the PNAD 2004 data, the standard deviation of log earnings is 0.95 for adults between 32 and 65 years old, after controlling for state, race, and gender fixed effects.

Intergenerational correlation of earnings. The intergenerational correlation of earnings measures the persistence of earning between two generations of the same family. Specifically, it is the slope coefficient of regressing the log earnings of the old parent on the log earnings of her child.For Brazil, and for developing economies in general, however, there are no panel data that would allow for such an estimation to the best of our knowledge. Ferreira and Veloso (2006) overcome this challenge by estimating the intergenerational persistence of earnings in Brazil using a synthetic parents' income, inferred from parental education and occupation, similar to Angrist and Krueger (1992), Arellano and Meghir (1992), and Björklund and Jäntti (1997).[22] We calibrate our model using 2004 data; hence, young parents are 32-36 years old in 2004 and, therefore, were born in 1968-1972. For the cohort born in 1967-71—the closest to our calibrated time period—Ferreira and Veloso (2006) estimate intergenerational earnings persistence at 0.46.[23] In our model, parameter ρ drives the persistence in innate ability, and, in equilibrium, it also drives the persistence in earnings. To match the intergenerational correlation of earnings, we calibrate ρ at 0.30. This is consistent with the the estimation made by Holter (2015), as well as with the calibration of Restuccia and Urrutia (2004), and Blankenau and Youderian (2015).

Fraction of applications. Based on the Brazilian educational census, the ratio between applicants and public college spots is 7. Since we have a measure of 4.8% of public college spots, the measure of applicants is targeted at 34%. The noise of the admission exam (σ_p) directly affects this moment. For instance, if there is no noise in the admissions process and the admission skill is perfectly observed, then only students who are admitted would apply to college. Thus, we calibrate σ_p to match the measure of applicants at 0.34.

[22] This is a two-stage procedure using the Brazilian National Household Survey data. First, they create a subsample of parents (between 25-64 years old, working full time, and located in urban areas), and regress their income on education and occupation dummies, controlling for cohort fixed effects. In a second step, the first-stage regression is use to forecast the wages of the actual parents because in the survey data we observe parents education and occupation but not their wages. This procedure allows them to construct synthetic wages for parents given their occupation and educational background. Finally,they estimate intergenerational persistence of earnings for different cohorts by regressing log synthetic earnings of parents on log earnings of their child.

[23] See Table 15 in Ferreira and Veloso (2006). For the US, the intergenerational correlation of earnings is slightly slower at about 0.40, using the PSID data (Solon 1992).

Table 3: Calibration Targets

This table reports all calibration targets along with their respective parameters in the model. The first two columns provide the model parameters, and the last three columns provide their respective moment both in the data and in the model. The standard deviation of log earnings (row i) and the intergenerational correlation of earnings (row ii) use the Brazilian National Household Survey (PNAD 2004) data. The fraction of applicants is computed based on the 2008 report made by the National Institute for Educational Research at the Brazilian Ministry of Education (INEP/MEC 2008) using 2004/8 data. Educational expenditure moments (rows iv-v) use data from the Brazilian Household Budget Survey (POF 2004) and the World Bank (2004). Finally, we compute the college wage premium (row vi), the measure of students in private college (row vii), the fraction of low-income students in public college (row viii), and the fraction of students in public school (row ix) using PNAD 2004. A detailed description of the construction of these moments is provided in Section 4.1. Low-income students in row (viii) refer to those in the second quintile of the income distribution.

	Parameters		Moments	Data	Model
i	σ_π	0.78	Standard deviation of log earnings	0.95	0.94
ii	ρ	0.30	Intergenerational correlation of earnings	0.46	0.44
iii	σ_p	1.08	Fraction of applications	0.34	0.32
iv	a	8	College expenditure relative to GDP	0.014	0.013
v	ψ	0.15	Early education expenditure relative to GDP	0.060	0.060
vi	$\overline{p}$	2.74	College wage premium	2.68	2.66
vii	θ	0.74	Fraction of students in private college	0.12	0.12
viii	γ	0.78	Fraction of low-income students in public college	0.07	0.06
ix	α	0.54	Fraction of students in public school	0.87	0.88

College expenditure relative to GDP. To measure expenditures in college education relative to GDP in Brazil, we rely on two data sources. First, according to the Brazilian Household Budget Survey (POF)[24] for 2003-2004, the average fraction of after-tax income spent on education was 4.08%. Second, according to World Bank data, the average Brazilian tax revenue rate for the period was 15.9%, which means that 3.43% ($4.08 \times (1 - 0.159)$) of households' before-tax income was spent on education. Also according to the World Bank, the Brazilian government spent 4.01% of GDP on education in the same period. Hence, the total–households and government—expenditures in college education was 7.44% of total output (4.01% + 3.43%). Finally, according to the World Bank, 18.92% of total education expenditures are allocated to higher education in Brazil, and, therefore, the aggregate investments in education are 1.4% ($0.1892 \times 7.44\%$) in higher education and 6% ($(1 - 0.1892) \times 7.44\%$) in pre-college education relative to GDP. Hence, we match college education expenditures over GPD at 1.4% by varying the cost of college education (parameter a).

Early education expenditure relative to GDP. From the previous calculations using data from the World Bank for Brazil and the Brazilian Household Budget Survey, 1.4% of GDP is spent in higher education and 6% is spent in pre-college education. We target pre-

[24]Pesquisa de Orçamento Familiar.

college expenditures over GPD at 6% by varying the concavity of the human capital formation function (parameter ψ). From Equation (6), it is clear that as the parameter ψ increases, so does the return from pre-college education. This is true regardless of the college education obtained—no college or a private college or public college degree. Therefore, ψ affects pre-college educational investment of *every* agent investing in education, independently of their expected level of college education. As a result, a higher ψ leads to an increase in aggregate expenditures on early education.

College wage premium. We compute the college wage premium as the average wage conditional on having a college degree relative to not having a college degree. Formally, to estimate the college wage premium, we regress earnings for adults between 32 and 48 years old on a dummy for college degree, controlling for state, race and gender fixed effects. In 2004, the college wage premium was 2.68 for college graduates relative to those with only a high school degree. The public college human capital gain ($\overline{p}$) drives the college wage premium by increasing the earnings of college graduates from public college, so we calibrate $\overline{p}$ to match the college wage premium.

Fraction of students in private college. The measure of agents attending college depends on the human capital gains from a college degree. In the model, a key determinant of college gains is the share of non-college human capital relative to college human capital in the firm's production technology. The parameter θ drives the demand for skilled labor, equilibrium wages and, therefore, equilibrium human capital gains from college. When θ decreases, the weight on college-degree labor increases, which increases the demand for skilled labor, pushing up college degree wages and the supply of skilled labor. Because the public college spots are fixed, any change in the supply of skilled labor comes from private college education. As a result, we calibrate θ to match the measure of students in the private college.[25]

Fraction of low-income students in public college. To calibrate the elasticity of admission skill with respect to early educational investments (parameter γ), we target the fraction of public-school students in the 40^{th} percentile of the income distribution, which

[25]It is important to point out that θ might affect the college wage premium, as well. On the one hand, a lower θ increases wages, which might lead to a higher college wage premium. On the other hand, an increased pool of college students will lower the average human capital of college graduates by having more, arguably less-skilled, students in private college. More importantly, $\overline{p}$ does not affect the benefit of private college much because it primarily affects the earnings for those attending public college. As a result, we can calibrate θ and $\overline{p}$ to match the measure of students in the private college and the college wage premium.

is 7% for the Brazilian case.[26] Given that admission skill (z) is the main driver of college admissions probability (i.e., test scores), parameter γ reflects the sensitivity of the probability of college admission with respect to pre-college education. As a result, γ affects the returns of pre-college education on the unconditional expected future human capital exclusively by making the probability of college admission more responsive to pre-college educational investments. As a direct implication, college applicants on the margin of being admitted are the most affected by changes in γ, because additional investment in pre-college education may significantly improve their odds of a successful college application. For all the other agents, the returns to pre-college education do change, but minimally—e.g., those who do not even apply to college or those who apply and are already likely to be admitted.

The fraction of low-income applicants admitted is sensitive to γ precisely because it affects marginal applicants' investment decisions the most. Importantly, marginal applications from wealthier families benefit disproportionately more from a higher value of γ. Lower-income households have a higher marginal utility of consumption, and, therefore, small adjustments to their consumption-investment bundle are enough to optimally adjust their policy functions. Hence, higher γ means that higher-income applicants increase their investment in pre-college education more than lower-income households do. As a result, more low-income applicants are crowed out from being admitted to college, and fewer lower-income applicants are admitted.

Fraction of students in public school. The fraction of students in public high schools was 87% in 2004, and, in the model, this fraction is directly affected by public school inefficiency (parameter α). If α is low (high), then public school has a lower (higher) return and more (fewer) young parents would enroll their child in a private school. Thus, we calibrate α to match the fraction of young children in public schools.

Finally, in Appendix E, we further verify our calibration strategy by using the methodology developed by Daruich (2018) to highlight that each one of targeted moments from the data relates to a different parameter in the model. His methodology provides a clean and intuitive way to depict the sensitivity of model-implied moments with respect to specific parameters of the model. The general idea is to randomly simulate different parameterizations of a model and compute moments implied by each parameterization. Then, we show that each parameter directly relates to a specific model-implied moment while not being affected

[26] Using PNAD data, 7% of public college students are in the bottom 40% of the income distribution. To compute these moments of the college income distribution, we use the 2002 PNAD so that we do not include students in private college with a ProUni scholarship. The affirmative action policy to be implemented in Brazil focuses on the first quintile of the income distribution, for which we also have a good fit. The fraction of public college students in the bottom quintile is 2.8% in the data and 3.2% in the model. We further discuss this in Panel B of Table 2.

Table 4: Non-Targeted Moments

This table reports several moments from the data and their model counterpart. In Panel A, we report the percentile of the log income distribution relative to the median. In the model, this variable is $\log(wh_o)$. Alternatively, we have included income of the old child, as well, but the results would be similar. To compute these cross-sectional moments of the income distribution, we use family income reported in the Brazilian National Household Survey (PNAD 2004). In Panel B, we compare the income distribution of students admitted to public college relative to the overall population. In the model, we report the fraction of students coming from each income quintile. In the data, we follow the same reasoning and condition family income on students who are newly admitted to college; then we compute the fraction coming from each quintile.

	Data	Model
Panel A: Income Distribution, $\log(h_o)$		
prc 5 - prc 50	-1.36	-1.34
prc 10 - prc 50	-1.03	-1.03
prc 25 - prc 50	-0.55	-0.41
prc 75 - prc 50	0.59	0.72
prc 90 - prc 50	1.25	1.33
prc 95 - prc 50	1.69	1.82
Gini Index	0.524	0.526
Panel B: Fraction of College Students from Different Income Quintiles		
Fraction from income quintile 1	0.02	0.02
Fraction from income quintile 2	0.05	0.04
Fraction from income quintile 3	0.09	0.06
Fraction from income quintile 4	0.18	0.13
Fraction from income quintile 5	0.65	0.75

by other parameters.

4.2 Other non-targeted moments

In this section, we verify how our model performs in terms of matching non-targeted moments that are directly linked to our economic mechanism. As the main policy we study is income-based affirmative action, it is crucial that our calibrated model matches the income distribution.

First, we verify that the model matches other moments of the income distribution and the overall level of inequality. In the first panel of Table 4, we report different percentiles of income distribution relative to the median income. The model successfully replicates the cross-sectional distribution of income. For instance, the log income of the 90^{th} percentile relative to the median is 1.25 in the data and 1.33 in the model. At the bottom of the distribution, the model mirrors the data more closely. The log income of the 5^{th} percentile relative to the median are -1.36 and -1.34 in the data and in the model, respectively. The log income of the last decile relative to the median is -1.03 both in the data and in the model. The model also implies a Gini index close to the index observed empirically, with

0.524 being estimated from the data and 0.526 in the model.

In addition to the unconditional income distribution, we also verify whether the model matches the income distribution among public college students. The unconditional income distribution and the distribution conditional on attending public college are strikingly different. This is directly related to our mechanism because affirmative action policies directly affect the income distribution of college students. As part of our calibration exercise, we match the fraction of students from the bottom 40^{th} percentile—see row (viii) in Table 3. However, we do not attempt to match other moments of that distribution. For example, in the data, 65% of students starting public college are from the top quintile of the income distribution, while only 2% are from the bottom quintile. In the model, we find similar fractions at 75% and 2%, respectively. The second panel of Table 4 reports the fraction of students from each quintile, and, although not directly targeted by our calibration, the model closely matches the data counterparts.

5 Policy evaluation

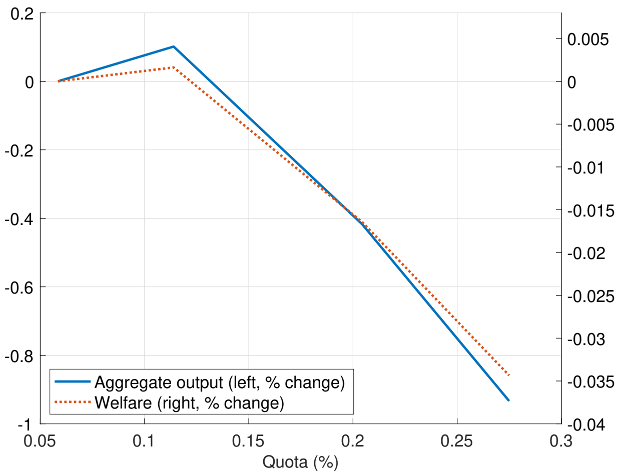
In this section, we describe the effects of income-based affirmative action, by comparing the steady-state equilibrium of our calibrated model with and without such policies.[27] We study the implications of income-based affirmative action policies for aggregate output, intergenerational correlation of earnings, and welfare. Welfare is measured as a population-weighted average of agents' value functions. Aggregate output measures the efficiency of college admissions because it is equal to the aggregate value of human capital in the economy. Thus, both welfare and aggregate output are important measures of the economic performance of income-based affirmative action policies. In addition, persistence of earnings evaluates whether the policy effectively promotes educational opportunities for socially-disadvantaged students.

Section 5.1 describes the trade-offs involved in favoring low-income applicants in college admissions. Section 5.2 discusses two distinct optimal policies: one that maximizes aggregate output and another that maximizes welfare. Finally, Section 5.3 evaluates the affirmative action currently being implemented in Brazil, and Section 5.4 presents several robustness exercises.

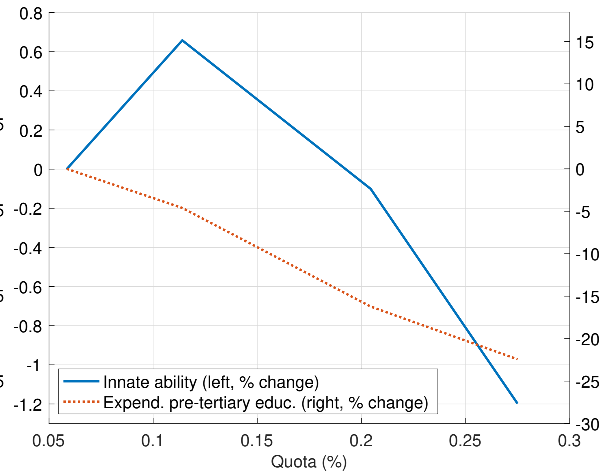
[27] In Appendix F, we numerically solve for the transition dynamics.

Figure 2: Affirmative Action for Low Income

This figure reports the effects of admissions quota policies for college applicants based on their income. The policy targets the bottom 40^{th} percentile of the income distribution. Panel A plots the effects on aggregate output (left axis) and welfare (right axis) against the intensity of the policy—i.e., the fraction of students in the preferentially-treated group attending public college. Panel B plots the implications for the average innate ability (left axis) and average expenditures on pre-college education (right axis) among public college students.



(a) Aggregate variables



(b) Averages conditional on public college

5.1 Understanding the trade-offs

One of the main reasons that income-based affirmative action policies in college education are a controversial topic is that they are essentially associated with an economic trade-off. While such policies may level the playing field and admit students with relatively high college returns, who otherwise wouldn't be admitted, income-based affirmative action may discourage investments from non-beneficiaries. Figure 2 shows this trade-off in our calibrated model. We simulate an income-based affirmative action policy benefiting applicants in the bottom 40^{th} percentile of the income distribution. Panel A displays the effects on aggregate output and welfare for different levels of quotas for the preferentially-treated group. Panel B plots the average innate ability and the average expenditures on pre-college education among students admitted to public college, for different levels of quotas for the preferentially-treated group.

For relatively low levels of the policy, aggregate output and social welfare increase, reaching a peak when the mass of eligible applicants who are admitted to public college doubles from its benchmark value of 5.8% to 11%. From this point, output and welfare start falling if more spots are allocated to applicants in the first two income quintiles.

The blue curve in panel B shows the reason that output and welfare follow this inverse U-shaped curve. For low levels of the quota policy, the average innate ability of students admitted to public college increases. This happens because there is a proportion of high-

Table 5: Optimal Affirmative Action

This table reports the effect of optimal affirmative action policies on persistence of earnings, aggregate output, and welfare. Column (1) reports the result for the affirmative action policy that maximizes aggregate human capital. Column (2) reports the affirmative action policy that maximizes welfare without decreasing aggregate human capital. We numerically solve for the optimal policies based on income quintiles. The effects are reported in basis points change relative to an economy without affirmative action.

	Max. Efficiency (1)	Max. Welfare (2)
Aggregate output change (bps)	23	10
Welfare change (bps)	2	3
Persistence of earnings change (bps)	-213	-574

ability students living in poor households who don't have funds to invest in education. High-income households, some of which have children with relatively low innate abilities, make education expenditures to increase their children's likelihood of admission, preventing high-ability, low-income students from being admitted. An income-based affirmative action policy remediates this and improves admissions by substituting students with relatively low innate abilities for higher-ability eligible students. However, as the quota policy intensifies, it starts admitting eligible students with lower innate abilities, crowding out higher-ability students and leading to a fall in aggregate output.

Lower educational investments from the non-preferentially-treated students is an equilibrium backlash of the policy, resulting in lower aggregate output as policy intensity increases. These graphs show that investments in pre-college education decline with policy intensity; however, they do not decrease much when the policy magnitude is close to zero. Therefore, a small-scale policy can improve admissions efficiency and increase aggregate output, without reducing educational investments much.

5.2 Optimal income-based affirmative action

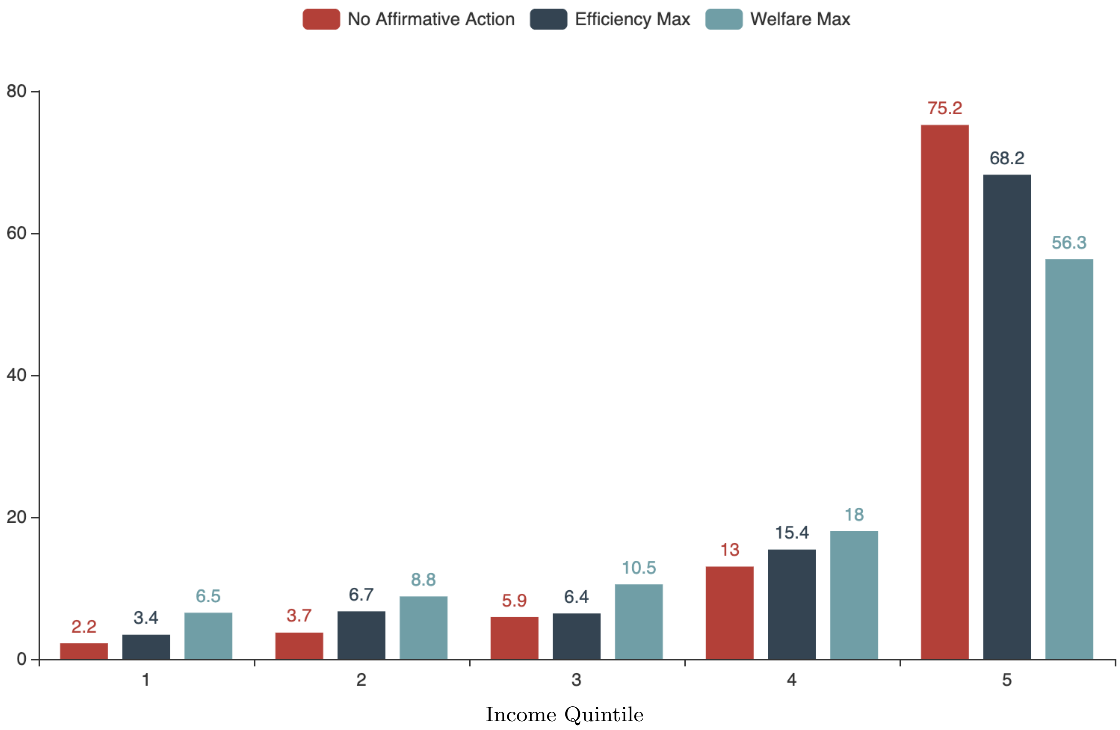
We numerically compute two optimal income-based affirmative action policies. First, we solve for the income-based affirmative action policy that maximizes aggregate output. Second, we compute the policy that maximizes welfare. We solve for optimal policies targeting different income quintiles with different quotas, allowing the policy maker to observe which income quintile applicants belong to, but not the income itself.[28]

Table 5 reports the results of our numerical optimization. In Column (1), we report

[28]The main reason for restricting the policy to condition on income quintiles is to keep the numerical optimization problem computationally less intense.

Figure 3: Income Distribution and Affirmative Action

This figure reports the fraction of students attending public college from each income quintile. We plot the income distribution under three different scenarios: without affirmative action, efficiency-maximizing affirmative action, and welfare-maximizing affirmative action.



the implications of the optimal efficiency-maximizing policy for aggregate output, welfare, and intergenerational persistence of earnings. The efficiency-maximizing affirmative action increases aggregate output by 23 basis points, while decreasing intergenerational persistence of earnings by 2.13%. This policy improves welfare, measured as the sum of all agents' utility. Column (2) shows the policy that maximizes welfare. Such a policy improves welfare by 3 basis points and decreases persistence of earnings by 5.74%. In summary, income-based affirmative action can be an efficiency-enhancing policy that improves welfare, and it can also be a powerful tool to promote educational opportunities at the aggregate level, effectively breaking persistence of earnings between the different generations of the same family.

The optimal policies implement different quotas based on the income quintile of the applicant. Figure 3 compares the income distribution of applicants admitted to public college under the optimal policies. Both optimal policies significantly admit more students from the lower quintiles and fewer applicants from the top quintiles. The efficiency-maximizing policy is more moderate, with a lower quota allocated to low-income applicants in order to improve the pool of admitted students without disincentivizing educational investments.

Table 6: Welfare measures

This table reports two welfare measures for the efficiency-maximizing and the welfare-maximizing policies. The first is the measure of households that are better off in a given counterfactual policy (unconditional and conditional on income quintiles). The second is calculated based on the proportion of consumption a household would be willing to pay in all periods of time and states of nature to have a given policy implemented. We report the unconditional average willingness to pay across all households and the averages by income quintiles.

	Max. Efficiency		Max. Welfare	
	Better-off households (%)	Willingness to pay (%)	Better-off households (%)	Willingness to pay (%)
Average	78	0.02	79	0.07
Quintile 1	97	0.18	98	0.55
Quintile 2	95	0.29	95	0.64
Quintile 3	95	0.13	97	0.53
Quintile 4	93	0.23	94	0.56
Quintile 5	10	-0.69	12	-1.90

In contrast, the welfare-maximizing policy is more intense, targeting the families with the highest marginal utilities. Both optimal policies change college demographics significantly. For example, the efficiency-maximizing policy increases the number of public college students from the second quintile of the income distribution by 80%, from 3.7% to 6.7%, while the welfare-maximizing policy increases the number of public college students from the bottom quintile by a factor of three, from 2.2% to 6.5%. In both policies, students in the top quintile lose their spots so that eligible students are benefited.

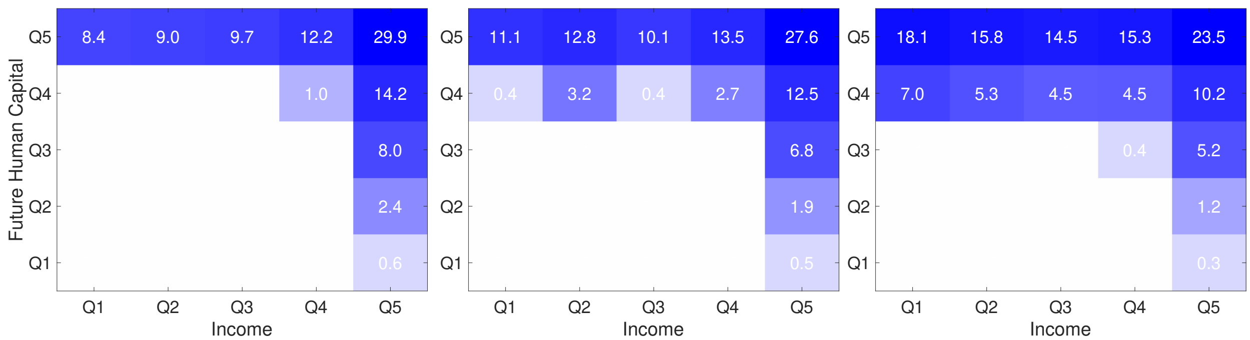
To further understand the welfare effects of the optimal policy interventions, Table 6 reports two additional welfare measures associated with each policy. The first measure is the fraction of households that are better off under a given affirmative action policy. The first column shows that 78% of the households prefer the efficiency-maximizing policy over the benchmark equilibrium, while 79% prefer the welfare-maximizing policy over having no intervention. The welfare gains are significantly heterogeneous across households: while approximately 95% of households in the first four income quintiles are better off under either of the optimal policies, only 11% of households in the top quintile would gain from the policies being implemented.

The second welfare measure is the willingness to pay. We measure how much each household would be willing to pay in terms of its consumption in all time periods and states of nature to have a given policy implemented.[29] The second column shows that

[29] See Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for more details on calculating willingness-to-pay measures in a similar environment.

Figure 4: Probability of Attending Public College

This figure reports the probability of attending public college by income and future human capital quintile. We define future human capital as the human capital a student would have if she were admitted to public college. Panel (a) reports the probabilities with no affirmative action in place. Panels (b) and (c) report these probabilities under the welfare- and efficiency-maximizing policies, respectively. Each panel is a heatmap with the probabilities reported in each quintile combination. For readability of the graph, we did not report probabilities below 0.2%.



(a) No Affirmative Action (b) Efficiency-max policy (c) Welfare-max policy

households in the first four income quintiles would be willing to pay, on average, 0.2% of their consumption in all periods and contingencies to have the efficiency-maximizing policy implemented. Households in the top quintile, however, would have to have their consumption levels increased by 0.69%, on average, to be compensated for their losses.

Although there are no significant welfare differences between the two policies if one uses the first welfare measure, the willingness to pay shows that the welfare-maximizing policy leads to even bigger and more heterogeneous welfare impacts. Households in the first four income quintiles, on average, would be willing to pay 0.57% of their consumption to have the welfare-maximizing policy implemented, while those in the top quintile would require their consumption to be increased by 1.9% in all periods and contingencies to be compensated for their losses.

To further illustrate how the optimal income-based affirmative action policies target income groups differently, we compute the probability of attending public college conditional on income and on future human capital quintile. We plot a heatmap of these probabilities in Figure 4. Panel (a) contains the heatmap of probabilities under no affirmative action. The average probability of attending public college for an applicant at the top income quintile and top human capital quintile is 30%, whereas an applicant in the same human capital quintile but at the bottom of the income distribution has only an 8.4% probability of attending public college. Strikingly, for the highest income quintile, even applicants with below-median human capital have a chance of attending public college. This highlights how income—and pre-college educational investments associated with it—can distort the pool of admitted students. Panels (b) and (c) report the heatmaps containing the probability of attending public college under the optimal efficiency- and welfare-maximizing policies, respectively. Both policies benefit the lower-income quintile the most, but at different intensities.

There are two key forces at play in shaping these optimal policies. On the one hand, they target low-income families the most, benefiting the most-constrained applicants. By segmenting the admissions' market by income groups, these policies roughly control for educational investments in early education. Although families' investments in pre-college education may still distort the pool of admitted students within each income group, this is less of an issue for low-income applicants given that they are more credit-constrained. The policies not only benefit applicants with higher human capital gains from college education but also take into account their effects on the human capital accumulation of future generations. Innate ability has some intergenerational persistence, and, by admitting low-income high-innate-ability applicants, optimal affirmative action policies induce social mobility for high-innate-ability families. Such social mobility allows future generations to enjoy a less credit-constrained environment, fostering future educational investments in high-innate ability-children. The optimal affirmative action policies effectively mitigate poverty traps, in which there could be generations of high innate ability families unable to invest in early education. On the other hand, the optimal policies are moderated in their intensities in order not to distort investment decisions. High-income families have the resources to invest in early education and the optimal policies keep these investments' incentives aligned.

The main difference between the welfare- and efficiency-maximizing policies is their intensity. The welfare-maximizing policy benefits low-income applicants more because they have high marginal utility and, therefore, the highest welfare gains from attending college. The efficiency-maximizing policy also benefits low-income applicants, but not at the same intensity as it trades off efficiency gains associated with higher aggregate output against welfare gains.

Evaluating the transition between steady-state equilibria The policy evaluation discussed above follows from comparing two steady-state equilibria: one with no policy in place and another with income-based affirmative action in college admissions. While this comparison allows us to infer the policy's long-term implications, it abstracts away from the transition between steady states. We present the transition dynamics of the efficiency-maximizing policy in Appendix F. The main takeaway from this exercise is that aggregate output initially decreases and then increases, converging to the new steady-state within a few generations. The welfare initially increases significantly and then continues to converge to its steady-state value.

Table 7: Long-run effects of the National Law of Quotas in Brazil

This table reports the effects of the affirmative action policy implemented in Brazil starting in 2012. Panel A shows the percentage changes in aggregate output, aggregate welfare, and intergenerational persistence of earnings. Panel B shows two welfare measures associated to the policy. The first is the proportion of better-off households under the policy (unconditional and conditional on income quintiles). The second welfare measure shows how much households would be willing to pay in terms of consumption in all time periods and contingencies to have the policy implemented (average willingness to pay across all households and conditional on income quintiles).

Panel A: Aggregate variables (% change)		
Aggregate output		-1.57
Welfare		-0.05
Persistence of earnings		-6.59
Panel B: Welfare measures (%)		
	Better-off households	Willingness to pay
Average	23	-0.08
Quintile 1	65	1.39
Quintile 2	19	-0.13
Quintile 3	13	-0.18
Quintile 4	12	-0.35
Quintile 5	6	-1.08

The policy, as soon as it is implemented, successfully changes college demographics by admitting more low-income applicants with higher returns to college education and fewer high-income applicants with lower returns to college education. On the one hand, when the policy is implemented, it benefits the most the families in lower-income quintiles, leading to a large initial welfare gain because of these families higher marginal utility. On the other hand, upon implementation of the policy, there is a decline in the quality of non-college educated workers leading to a decrease in aggregate unskilled human capital and, therefore, a decrease in aggregate output. As new generations arrive, families adjust their investment decisions and the economy converges to the new steady state with higher aggregate output and higher welfare.

5.3 Evaluating affirmative action in Brazil

In 2012, the Brazilian congress approved the major-specific quota policy targeting low-income and public school students. Over the subsequent three years, the government gradually implemented the policy in all federally-sponsored universities. The law says that 50% of students admitted to each degree program in Federal Universities must have studied in a public high school. Furthermore, half of those spots are reserved for low-income applicants—

i.e., those with a family income less than 1.5 times the minimum wage, who are those in the first quintile of the income distribution. Finally, a fraction of the spots for low-income applicants are reserved for non-white candidates, in line with the racial distribution in the university's geographical location.[30]

We use our calibrated model to shine a light on the long-run implications of the affirmative action policy implemented in Brazil. Formally, we evaluate a policy that considers the number of public-college spots as fixed and allocates 50% of those spots to applicants from public schools, with half of those spots, reserved for applicants in the first quintile of the income distribution.

Table 7 shows that the Brazilian policy decreases long-run output by 1.57% and the population-weighted sum of all agents' value function (welfare) by 0.05%. In line with its objectives, the policy promotes educational opportunities by decreasing the intergenerational persistence of earnings by 6.59%.

The Brazilian policy benefits 23% of all households, with the majority of better-off households located in the first income quintile. The average willingness to pay for the policy is of -0.08%, and the gains of the first income quintile are significant: those households would be willing to pay 1.39% of their consumption in all periods and contingencies of the policy steady state to have the quotas in place.

Brazil's policy can be improved. According to our previous analysis, a better-designed policy could increase welfare, increase aggregate output and break the intergenerational persistence of earnings.

5.4 Robustness

In this section, we show that income-based affirmative action in college admissions still improves welfare and increases aggregate output under variations of our benchmark model and calibration. We conduct several robustness exercises.

Taken together, these exercises suggest that the economic mechanism behind our results, discussed in detail in Section 3.3, is not a feature of our modeling or calibrating choices, but a robust consequence of credit constraints and private investments in education.

[30]Federal law 12.711, August 29, 2012. Although the law was approved in 2012, in 2005, some universities had already started to implement affirmative action policies in a decentralized way. For example, the Federal University of Bahia (UFBA) introduced a similar quota system in 2005. After the implementation, the fraction of students from public schools increased 50%, from 33.8% to 51%. In some selected majors, it increased more than 400%; for example, in architecture, the fraction of public school students increased from 10.7% in 2004 to 43.7% in 2005.

Table 8: Robustness analyses

This table reports the effect of optimal affirmative action policies on aggregate output and welfare. We numerically solve for the optimal affirmative action under different model specifications. Column (1) describes the alternative specifications—See Section 5.4 for details. Columns (2)-(3) report the effects of the optimal affirmative action policy on aggregate output and welfare, under the efficiency-maximizing policy. Columns (4)-(5) report the effects under the welfare-maximizing policy. The effects are reported in basis points change relative to the same economy without affirmative action.

	Efficiency-maximizing policy		Wefare-maximizing policy	
Model Variation	Output gain	Welfare Gain	Output gain	Welfare Gain
(1)	(2)	(3)	(4)	(5)
Households pay public college tuition	23	6	21	6
More public college spots	37	3	37	3
Calibration with $\alpha = 1$	8	1	-8	2
College dropout risk	34	6	34	6
Imperfect substitution btw. e and g	20	1	-4	6

Households pay public college tuition In many countries, including the U.S., students must pay college tuition regardless of the college's ownership type (public or private). We solve our model for a world in which the public college student has to bear the entire cost of a college education. In this case, all labor taxes collected by the government are invested in pre-tertiary education, and the budget constraint of the old parent when the old child goes to college (equation 13) becomes

$$c_o + a\eta = (1-\tau)\left[\left(d_c w_c + (1-d_c)w_n\right)h_o + w_c h'_y\left(1-\eta-b\right)\right]$$

The first row of Table 8 shows the effects of the efficiency- and the welfare-maximizing policies on aggregate output and welfare. Compared to the baseline optimal policies, now the effects on aggregate variables are now even stronger.

More public college spots Recently, there has been an increase in the number of public college spots at government-sponsored Brazilian universities. We compute the optimal policies for an alternative benchmark equilibrium in which the number of public college spots is 10% higher (e.g., we set $\mathcal{S} = 0.0528$). We obtain a result similar to that in the previous robustness exercise, with aggregate output and welfare increasing more under the optimal policies, compared to in the case in which $\mathcal{S} = 0.048$.

Calibration with $\alpha = 1$ An essential characteristic of the Brazilian educational environment is that public schools have lower quality, on average, than private schools. We simulate the model assuming that $\alpha = 1$, meaning that public schools and private schools have the same quality. We find that the efficiency-maximizing affirmative action policy increases output by 8 basis points and welfare by 1 basis point, while the welfare-maximizing policy increases welfare by 2 basis points, but aggregate output falls by 8 basis points.

College dropout risk To evaluate if our findings are robust to having college dropouts in the model, we modify our framework in two dimensions. First, we assume that, if a student drops out of college, forgone earnings equal two years of labor earnings (instead of four years when the students complete their college education). Second, based on the data, we assume that the probability of a student dropping out of college (public or private) is a function of her parent's income:

$$\min\{\kappa_1[1 + (d_c w_c + (1 - d_c) w_n) h_o]^{\kappa_2}, 1\}.$$

This functional form follows Restuccia and Urrutia (2004). The parameter κ_1 controls the overall dropout probability, and κ_2 determines the correlation between dropping out and parent's income. Low-income students in developing countries have higher incentives than high-income students to drop out in order to join the labor market and contribute to their households' earnings.

We use 2004 PNAD data on family income and college dropping out to calibrate parameters κ_1 and κ_2. Table 9 shows the moments in the data in the model. We set $\kappa_1 = 0.7$ for the model to fit a proportion of dropout equal to 16%, and $\kappa_2 = -0.5$ so that the model reproduces the negative correlation between family income and dropping out found in the data. Table 8 shows that the efficiency- and the welfare-maximizing policies are similar and increase output and welfare by 34 and 6 basis points, respectively.

Imperfect substitution between e and g We now allow for private and public expenditures to be imperfect substitutes in the formation of an agent's human capital. The effective education investment of a student who attends public school is given by

$$\hat{e} = \alpha \left(e^\nu + g^\nu\right)^{1/\nu},$$

where $\nu \in (0, 1]$ determines the degree of substitutability between e and g. We set $\nu = 0.7$ to allow for imperfect substitutability, and we find that the optimal income-based affirmative action policies remain qualitatively unchanged under this new parametrization. Aggregate output increases by 20 basis points in the efficiency-maximizing policy, while social welfare

Table 9: College dropout calibration

This table shows moments related to dropouts in the data and in the extended model that includes dropouts. See subsection "College dropout risk" in Section 5.4 for more details. Moments in the data are calculated from the 2004 PNAD.

Moments	Data	Model
Fraction of dropouts (%)	16	16
Frac. dropouts by income quintile (%)		
Quintile 1	42	44
Quintile 2	37	35
Quintile 3	29	29
Quintile 4	24	24
Quintile 5	13	15

is improved by 6 basis points in the welfare-maximizing intervention.[31]

5.4.1 Credit Constraint

Our model features no credit market between periods. We want to emphasize that each period in our model represents approximately 16-18 years and there is a perfect intraperiod credit market as agents optimize over one single budget constraint each period. Student loans are uncommon and do not constitute a large market in Brazil. However, we stress that this is a reasonable assumption even for the US, given that the vast majority of student loans in the US have repayment term of 10 years (Avery and Turner 2012, Lochner and Monge-Naranjo 2016, Mueller and Yannelis 2018). In addition, not every student takes on student loans. In fact, according to the National Center for Education Statistics for the 2007-08 cohort, about 34.1% of students did not take on student loans. However, students who do take on loans can renegotiate their debt, effectively extending their repayment period. For the 1995-1996 cohort of college students in the US, out of those who took on student loans, the median percent of the amount owed 20 years after entering postsecondary education was 5% for students that attained a bachelor's degree or higher, while the unconditional median was 22%.[32]

Therefore, the bulk of student debt is paid within a 20-year window: the median borrower paid nearly 80% of the loan, while the median borrower who attained a bachelor's degree or

[31] If we vary ν from 0.5 to 0.9, which implies an elasticity of substitution varying from 2 to 10, we find similar results.

[32] See Tables 2 (Loan Amount Owed) and 15 (Education Loans) from NCES:
https://nces.ed.gov/Datalab/TablesLibrary

higher paid 95%. This is consistent with the findings by Lochner and Monge-Naranjo (2014). Using data from the 2003 Baccalaureate and Beyond Longitudinal Studies, the authors document a 8.3% non-payment rate and a 5.2% fraction of debt owed among borrowers 10 year after their graduation. In practice, there is also debt forgiveness for some non-profit and government employees, which may lead to long-term strategic default and inflate these statistics. In our model, all loan debt should be paid in a 18-year period, which is not far from the evidence based on US data. In addition, in order to have affirmative action policies to improve efficiency, it is enough that credit conditions vary with parents' income. As long as low-income families face more credit constraints, which is a reasonable assumption, our economic mechanism showing efficiency gains from affirmative action remains unchanged. College credit conditions varying by income is in line with the findings discussed by Lochner and Monge-Naranjo (2012).

6 Concluding remarks

In this paper, we evaluate the effects of income-based affirmative action policies on welfare, efficiency, and intergenerational correlation of earnings. Using microdata at the student level from Brazil, we first find that low-income students outperform their higher-income peers in college when they have the same college-admission score. This suggests efficiency gains can be obtained if college admissions favor low-income applicants. We investigate whether the efficiency and welfare gains generated by income-based affirmative action policies are economically meaningful and outweigh unintended distortions in educational investments. To that end, we build an overlapping-generations model with heterogeneous agents and calibrate it using data from Brazil.

Our main results are that (i) income-based affirmative action can promote economic efficiency and increase aggregate output by improving the pool of students admitted to public college; and that (ii) an optimally-designed affirmative action policy benefits nearly 80% of the families and reduces intergenerational persistence of earnings by 5.7%. Therefore, affirmative action in college admissions is an effective policy to improve welfare and to promote educational opportunities for socially disadvantage students.

Given the significant welfare gains associated with an optimal policy, it might seem surprising that income-based affirmative action policies are politically controversial. Our results shed light on this apparent paradox from two different angles. First, our results show that—even for the optimal income-based affirmative action—households on the top quintile of the income distribution would significantly oppose the policy. This resistance might pose challenges when trying to pass such policies through the political process. Second, the design of an optimal income-based affirmative action policy is complicated, and, as a result, a sub-optimal policy may be implemented. When evaluating the policy implemented in Brazil, we show that a significant majority of households in the top four quintiles of the income distributions are made worse off by the policy.

References

Angrist, J. D., and A. B. Krueger (1992): "The effect of age at school entry on educational attainment: an application of instrumental variables with moments from two samples," *Journal of the American statistical Association*, 87(418), 328–336.

Arcidiacono, P. (2005): "Affirmative Action in Higher Education: how do admission and financial aid rules affect future earnings?," *Econometrica*, 73(5), 1477–1524.

Arellano, M., and C. Meghir (1992): "Female labour supply and on-the-job search: an empirical model estimated using complementary data sets," *The Review of Economic Studies*, 59(3), 537–559.

Arenas, A., C. Calsamiglia, and A. Loviglio (2020): "What Is at Stake without High-Stakes Exams? Students' Evaluation and Admission to College at the Time of COVID-19," Discussion Paper 13838, IZA Institute of Labor Economics.

Assunção, J., and B. Ferman (2013): "Does affirmative action enhance or undercut investment incentives? evidence from quotas in Brazilian public universities," Working paper.

Avery, C., and S. Turner (2012): "Student Loans: Do College Students Borrow Too Much–Or Not Enough?," *Journal of Economic Perspectives*, 26(1), 165–92.

Becker, G. S., and N. Tomes (1979): "An equilibrium theory of the distribution of income and intergenerational mobility," *The Journal of Political Economy*, pp. 1153–1189.

Ben-Porath, Y. (1967): "The production of human capital and the life cycle of earnings," *The Journal of Political Economy*, pp. 352–365.

Björklund, A., and M. Jäntti (1997): "Intergenerational income mobility in Sweden compared to the United States," *The American Economic Review*, 87(5), 1009–1018.

Blankenau, W., and X. Youderian (2015): "Early childhood education expenditures and the Intergenerational persistence of income," *Review of Economic Dynamics*, 18, 334–349.

Chan, J., and E. Eyster (2003): “Does banning affirmative action lower college student quality?,” *The American Economic Review*, 93(3), 858–872.

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2007): “A Quantitative Theory of Unsecured Consumer Credit with Risk of Default,” *Econometrica*, 75(6), 1525–1589.

Coate, S., and G. C. Loury (1993): “Will affirmative-action policies eliminate negative stereotypes?,” *The American Economic Review*, 83, 1220–1240.

Daruich, D. (2018): “The macroeconomic consequences of early childhood development policies,” *FRB St. Louis Working Paper*, (2018-29).

Ebenstein, A., V. Lavy, and S. Roth (2016): “The Long-Run Economic Consequences of High-Stakes Examinations: Evidence from Transitory Variation in Pollution,” *American Economic Journal: Applied Economics*, 8(4), 36–65.

Epple, D., R. Romano, and H. Sieg (2008): “Diversity and affirmative action in higher education,” *Journal of Public Economic Theory*, 10(4), 475–501.

Estevan, F., T. Gall, and L.-P. Morin (2022): “Redistribution without Distortion: Evidence from An Affirmative Action Programme At a Large Brazilian University,” *Forthcoming in The Economic Journal*.

Fang, H., and P. Norman (2006): “Government-mandated discriminatory policies: theory and evidence,” *International Economic Review*, 47(2), 361–389.

Ferreira, S. G., and F. A. Veloso (2006): “Intergenerational mobility of wages in Brazil,” *Brazilian Review of Econometrics*, 26(2), 181–211.

Fryer, R., and G. Loury (2007): “Valuing identity: The simple economics of affirmative action policies,” *Forthcoming in Journal of Political Economy*.

Gordon, G. (2019): “Efficient Computation with Taste Shocks,” Working Paper 19-15, Federal Reserve Bank of Richmond.

Havranek, T., Z. Irsova, L. Laslopova, and O. Zeynalova (2020): “The Elasticity of Substitution between Skilled and Unskilled Labor: A Meta-Analysis,” *Working Paper*.

Hickman, B. R. (2010): “Effort, race gaps and affirmative action: A game-theoretic analysis of college admissions,” *Typescript, University of Iowa Department of Economics*.

——— (2013): "Pre-college human capital investment and affirmative action: a structural policy analysis of us college admissions," Working paper.

Holter, H. A. (2015): "Accounting for Cross-Country Differences in Intergenerational Earnings Persistence: The Impact of Taxation and Public Education Expenditure," *Quantitative Economics, forthcoming.*

INEP (2008): "Resumo Ténico: Censo da Educação Superior 2008," Discussion paper, Ministério da Educação (Brasil), Instituto de Estudos e Pesquisas Educacionais Anísio Teixeira (INEP).

Kapor, A. (2015): "Distributional Effects of Race-Blind Affirmative Action," Working paper.

Krishna, K., and V. F. Robles (2012): "Affirmative Action in Higher Education in India: Targeting, catch up, and Mismatch," NBER Working Paper No. w17727.

Krishna, K., and A. Tarasov (2013): "Affirmative Action: One size does not fit all," Working paper.

Lochner, L., and A. Monge-Naranjo (2012): "Credit constraints in education," *Annu. Rev. Econ.*, 4(1), 225–256.

——— (2016): "Student loans and repayment: theory, evidence, and policy," in *Handbook of the Economics of Education*, vol. 5, pp. 397–478. Elsevier.

Lochner, L. J., and A. Monge-Naranjo (2014): "Default and repayment among baccalaureate degree earners," Discussion paper, National Bureau of Economic Research.

Mello, U. (2022): "Centralized Admissions, Affirmative Action and Access of Low-income Students to Higher Education," *Forthcoming in American Economic Journal: Economic Policy.*

Moro, A., and P. Norman (2003): "Affirmative action in a competitive economy," *Journal of Public Economics*, 87(3), 567–594.

Mueller, H. M., and C. Yannelis (2018): "The rise in student loan defaults," *Journal of Financial Economics.*

Peruffo, M., and P. C. Ferreira (2017): "The long-term effects of conditional cash transfers on child labor and school enrollment," *Economic Inquiry.*

Restuccia, D., and C. Urrutia (2004): "Intergenerational persistence of earnings: The role of early and college education," *The American Economic Review*, 94(5), 1354–1378.

Solon, G. (1992): "Intergenerational income mobility in the United States," *The American Economic Review*, pp. 393–408.

Tauchen, G. (1986): "Finite state Markov-chain approximations to univariate and vector autoregressions," *Economics letters*, 20(2), 177–181.

Veloso, N. G. (2016): "The Impact of College Admissions Policies on The Performance of High School Students," Working paper.

Vieira, R. S., and M. Arends-Kuenning (2019): "Affirmative action in Brazilian universities: Effects on the enrollment of targeted groups," *Economics of Education Review*, 73, 101931.

Appendix

A Data sources, dataset construction, and variables definition

Our main data source is the IDD ("*Indicador de Diferença entre os Desempenhos Observado e Esperado*") dataset for the years 2014-2019.[33] The IDD is an index developed by the Ministry of Education, which aims to evaluate the quality of undergraduate programs in Brazil. It measures the college evolution of graduating students using as inputs ENEM grades (which proxy a student's quality upon entry) and the ENADE exam (which measures college performance).

The IDD dataset is built from a merge between the ENEM and ENADE microdata and contains ENEM and ENADE scores at the student level. However, it doesn't contain information regarding a student's family income. To have this information, we merge the IDD dataset with the ENADE microdata. Since IDD doesn't contain students' identifier numbers, we use the following combination of variables to identify a student in the two datasets: higher education institution, degree, municipality where the degree was taken, year when the undergraduate program started, and the ENADE score. To execute this merge, we first need to drop observations that cannot be uniquely identified by those variables from both datasets. In this stage, we drop 152,330 observations from the IDD dataset (11%), and 378,860 observations from the ENADE dataset (16%). We then merge the two datasets, and 97% of the observations in the IDD dataset are found in the ENADE dataset.

Below is a list with the definitions of the variables we use:

- **ENADE score.** Weighted average of general (25%) and field-specific (75%) scores. This weighting is used in the IDD index construction, and this weighted average comes pre-calculated in the dataset. ENADE is the exam that a student takes in the last year of its undergraduate studies.

- **ENEM scores.** Composed of four variables: mathematics, humanities, sciences, and languages scores. ENEM is the main college admission score in Brazil.

- **Higher education institution.** Numeric code identifying a higher education institution.

- **Gender.** Male/female. Self-reported.

- **Race.** White/black/pardo/Asian/indigenous. Self-reported.

[33] We downloaded the IDD microdata at INEP's microdata website (https://www.gov.br/inep/pt-br/acesso-a-informacao/dados-abertos/microdados) on December 2021.

- **Family earnings.** Total family earnings, including the student's. Categorical variable. Self-reported.

- **Work status.** Doesn't work/Eventually works/Works less than 20 hours per week/Works between 21 to 39 hours per week/Works more than 40 hours per week. Self-reported.

B Equivalence between quotas and bonus points

Proposition 1. *For every equilibrium with quotas in the admissions process, there is an equilibrium with bonus points and no quota with the same allocation. The converse is also true: for every equilibrium with bonus points in the admissions process, there is an equilibrium with quotas and no bonus points with the same allocation.*

Proof:

First, let us take an equilibrium in which bonus points of p_{bonus} are implemented for a certain treatment group. Let $\bar{q}^*$ and g^* be the grade point cutoff and the government spending on early education in such an equilibrium. Furthermore, let SP_{pt} be the fraction of public college students in the preferential treatment group.

For a given admission skill level z, a student in the preferential treatment group will be admitted to a public college with a probability given by

$$q^{\text{bonus}}\left(z, \bar{q}^*\right) = 1 - \Phi\left(\frac{\log\left(\bar{q}^*\right) - \log\left(z\right) - \log\left(p_{bonus}\right)}{\sigma_P}\right)$$

If the students is not in the treatment group, her probability of admission is

$$q^{\text{no bonus}}\left(z, \bar{q}^*\right) = 1 - \Phi\left(\frac{\log\left(\bar{q}^*\right) - \log\left(z\right)}{\sigma_P}\right)$$

To find the equilibrium with quotas, take the same allocation, but there will be two grade point cutoffs: one for the quota ($\bar{q}^{\text{quota}}$) and another for the no-quota student ($\bar{q}^{\text{no quota}}$). These cutoffs are defined as $\log\left(\bar{q}^{\text{quota}}\right) = \log\left(\bar{q}^*\right) - \log\left(p_{bonus}\right)$ and $\log\left(\bar{q}^{\text{no quota}}\right) = \log\left(\bar{q}^*\right)$.

Using these cutoffs, the parent's maximization problem is exactly the same. Then, this allocation, along with the government expenditure on early education and the grade cutoffs, constitute an equilibrium. Similarly, from an equilibrium with quotas, the grade point cutoff gap between the preferential treatment group and the other students can be used to get the necessary bonus points to construct an equilibrium with bonus points that generates the same allocation.

C Theoretical Example

In this section, we present a theoretical example highlighting the main mechanism in our model. We show that affirmative action can increase the aggregate human capital, by improving the quality of admitted students. This example shares several features with the benchmark model discussed in the next section—in particular, the mismatch between agents' expected human capital and their likelihood of college admission. In this example, the aggregate human capital depends primarily on the correlation between an agent's ability and her likelihood of college admissions. However, college admission is based on educational investments and therefore wealthier agents endogenously crowd out low-income applicants from college admission. This example is designed to highlight that income-based affirmative action changes college demographics by admitting more low-income applicants with high expected human capital and less high-income applicants with lower expected human capital. Our full specification discussed in the next section relaxes several of these assumptions, while keeping this economic mechanism unchanged. All the detailed derivations of this example are discussed in Appendix C.1.

To keep the example tractable, we assume a continuum of agents with unit measure, and each agent is identified by an income-ability pair, i.e. (w, π). Income and ability are independent and identically distributed uniform random variables:

$$w \sim U(0,1), \qquad \pi \sim U(0,1), \qquad w \perp \pi.$$

Human capital depends on agents' ability and college education. Specifically, an agent's human capital is given by π if she does not attend college and $p\pi$ if she attends college, where $p > 1$ is the human capital boost from college.[34] An agent chooses whether to invest or not in education, $e \in \{0,1\}$, bearing an opportunity cost given by $\frac{e}{w}$. Investment in education affects an agent's expected human capital as it increases the probability of admission, namely Pr^{in}. An agent's maximization problem is given by:

$$\max_{e\in\{0,1\}} \left(1 - Pr^{in}\right)\pi + Pr^{in} p\pi - \frac{e}{w},$$

The likelihood of admission is given by $Pr^{in} = \frac{e}{\bar{q}}$. We motivate this functional form by assuming that each applicant takes an exam, with grade given by $q = \frac{e}{\varepsilon}$, where $\varepsilon \overset{i.i.d.}{\sim} U(0,1)$. We assume a fixed number of college spots, $\mathcal{S}$.[35] There is a grade-point cutoff, $\bar{q}$, such that

[34]In the main quantitative exercise we consider a more general human capital specification, based on Ben-Porath (1967).

[35]To ensure that at least a measure $\mathcal{S}$ of agents invest in education, we assume that: $1 - \mathcal{S} \geq \frac{1+\log(p-1)}{p-1}$. This assumption also ensures that $\bar{q} \geq 1$ and thus that $Pr^{in} \in [0,1]$. See details in

all applicants with grades above the cutoff are admitted to college. Hence, there is a unique equilibrium grade-point cutoff such that a measure $\mathcal{S}$ is admitted to college, clearing the admission's market.

Affirmative action based on income increases the grade of low-income applicants. Applicants with income below the threshold $T \in (0,1)$ have their grade given by $q = \frac{e}{\varepsilon}B$, where $B > 1$ is the grade bonus given by the policy. As a result, the probability of admission of beneficiaries of the policy becomes $Pr^{in} = \frac{e}{\bar{q}}B$. Therefore, the affirmative action is characterized by two parameters: T, defining the income group benefited by the policy, and B, defining its magnitude.

Under affirmative action, the solution to an agent's optimization problem is given by a threshold strategy:

$$e = 1 \Leftrightarrow \pi w > \begin{cases} \frac{\bar{q}}{(p-1)}, & \text{if } w > T \\ \frac{\bar{q}}{B(p-1)}, & \text{if } w \leq T. \end{cases}$$

An agent invests in education if, and only if, the product πw is sufficiently high. For each income level, there is an ability threshold such that an agent invests if her ability if sufficiently high.

In equilibrium, affirmative action has two distinct effects. First, there is a direct effect on low-income agents. The bonus point B directly increases the probability of admissions, for a given investment choice, and also incentivizes investment by lowering the $w\pi$ threshold required to invest. Second, there is an equilibrium effect through the grade-point cutoff $\bar{q}$ affecting all agents. The policy increases the cutoff $\bar{q}$, which decreases agents' probability of admission and disincentivizes investments by increasing the $w\pi$ threshold required to invest. The combination of these two effects changes college demographics by admitting more low-income applicants with high expected human capital and less high-income applicants with lower expected human capital.

To evaluate the effects of affirmative action, we first solve for the aggregate human capital, namely H, considering a policy of magnitude B.[36] Then, we focus on the marginal effect of the policy by calculating the derivative of the aggregate human capital with respect to the magnitude of the policy, evaluated at the absence of any policy. Specifically, we calculate $\frac{\partial H}{\partial B}\Big|_{B=1}$.

Appendix C.1.

[36] To solve for the aggregate human capital, we assume that the affirmative action policy targets an income group in which at least a positive measure agents invest in education: $T > \frac{\bar{q}_0}{p-1}$, where $\bar{q}_0$ is the admission's market clearing grade-point cutoff in the absence of any affirmative action policy. Specifically, $\bar{q}_0$ solves $\mathcal{S} = \frac{1}{\bar{q}_0} - \frac{1}{p-1} + \frac{1}{p-1}\log\frac{\bar{q}_0}{p-1}$. If $T \leq \frac{\bar{q}_0}{p-1}$, then at $B = 1$ affirmative action has no marginal effect, as no beneficiary of the policy invest in education.

The first step is to solve for the equilibrium grade-point cutoff. In equilibrium, for any policy, $\bar{q}$ is such that the measure of college spots equals the measure of admitted students, formally $\bar{q}$ clears the admissions' market:

$$\mathcal{S} = \int_0^1 \int_0^1 Pr^{in} \quad d\pi dw.$$

Solving the above expression and using the implicit function theorem, we obtain the marginal effect of increasing the policy's magnitude over the grade-point cutoff, evaluated at the absence of any policy:

$$\left.\frac{\partial \bar{q}}{\partial B}\right|_{B=1} = \bar{q}_0 \frac{T - \frac{\bar{q}_0}{p-1}}{1 - \frac{\bar{q}_0}{p-1}} > 0.$$

This means that the grade-point cutoff increases in the presence of affirmative action.

Finally, the marginal effect of the policy is given by:

$$\left.\frac{\partial H}{\partial B}\right|_{B=1} = \frac{1}{2T}\left(T - \frac{\bar{q}_0}{p-1}\right)(1-T) > 0.$$

Given that $T \in \left(\frac{\bar{q}_0}{p-1}, 1\right)$, affirmative action has a positive effect on aggregate human capital. All the detailed derivations of this example are discussed in Appendix C.1.

This example captures a key channel through which affirmative action increases aggregate human capital. The policy changes the composition of admitted students, by admitting more low-income applicants with high expected human capital and less high-income applicants with lower expected human capital. However, our example has simplifying assumptions. For instance, it is a static environment, in which income and ability are exogenous and independent. Also, investments in education are discrete and have no effect on human capital, except through the probability of admission; whereas ability has no effect on admissions, except through the investment threshold decision.

In the main text, we present a more comprehensive and realistic framework. We relax the above restrictions, and study affirmative action through the lens of an overlapping generations model. Our benchmark framework features a dynamic human capital formation process, in which educational investments affect human capital both directly, by increasing future earnings, and indirectly, by increasing the probability of college admission. Our model also considers public and private educational systems, endogenous opportunity cost of educational investments, and a tax system to finance public education.

C.1 Derivations

There is a continuum of agents with unit measure, and each agent is identified by an income-ability pair, i.e. (w, π). Income and ability are independent and identically distributed uniform random variables:

$$w \sim U(0,1), \qquad \pi \sim U(0,1), \qquad w \perp \pi.$$

As discussed in the paper, an agent's maximization problem is given by:

$$\max_{e \in \{0,1\}} \left(1 - Pr^{in}\right)\pi + Pr^{in} p\pi - \frac{e}{w},$$

where

$$Pr^{in} = \begin{cases} \frac{e}{\bar{q}}, & \text{if } w > T \\ \frac{e}{\bar{q}}B, & \text{if } w \leq T \end{cases},$$

T is the income threshold defined by the policy, B is the magnitude of the affirmative action policy, and $\bar{q}$ is the endogenous grade-point cutoff.

The optimal investment policy is given by:

$$e = 1 \Leftrightarrow \pi w > \begin{cases} \frac{\bar{q}}{(p-1)}, & \text{if } w > T \\ \frac{\bar{q}}{B(p-1)}, & \text{if } w \leq T. \end{cases}$$

We assume a fixed number of college spots, $\mathcal{S}$, and we assume that

$$1 - \mathcal{S} \geq \frac{1 + \log(p-1)}{p-1}.$$

This is coming the fact that if $B = 1$ and $\bar{q} = 1$, then

$$Pr(e = 1) = \int_0^1 \int_0^1 Pr^{in} \quad d\pi dw = \frac{1}{\bar{q}} \int_{\frac{\bar{q}}{p-1}}^1 \int_{\frac{\bar{q}}{w(p-1)}}^1 \quad d\pi dw = 1 - \frac{1 + \log(p-1)}{p-1}$$

Hence, if $1 - \mathcal{S} \geq \frac{1+\log(p-1)}{p-1}$, than $\bar{q} \geq 1$ and at least a measure $\mathcal{S}$ apply to college. This assumption also ensures that $Pr^{in} \in [0,1]$.

Next, we solve for the equilibrium grade-point cutoff for the case without affirmative action. In equilibrium, the cutoff $\bar{q}_0$ is such that the measure of college spots equals the measure of admitted students under $B = 1$:

$$
\begin{aligned}
\mathcal{S} &= \int_0^1 \int_0^1 Pr^{in} \quad d\pi dw \\
&= \frac{1}{\bar{q}_0} Pr\left(\pi w > \frac{\bar{q}_0}{p-1}\right) \\
&= \frac{1}{\bar{q}_0} \int_{\frac{\bar{q}_0}{p-1}}^1 \int_{\frac{\bar{q}_0}{w(p-1)}}^1 d\pi dw \\
&= \frac{1}{\bar{q}_0} \int_{\frac{\bar{q}_0}{p-1}}^1 1 - \frac{\bar{q}_0}{w(p-1)} dw \\
\therefore \mathcal{S} &= \frac{1}{\bar{q}_0}\left[1 - \frac{\bar{q}_0}{p-1} + \frac{\bar{q}_0}{p-1}\log\left(\frac{\bar{q}_0}{p-1}\right)\right].
\end{aligned}
$$

We also assume that $T \in \left(\frac{\bar{q}_0}{p-1}, 1\right)$, hence affirmative action policy will affect current applicants even if B is close to one.

Under affirmative action, if B is sufficiently close to one, then

$$
\begin{aligned}
\mathcal{S} &= \int_0^1 \int_0^1 Pr^{in} \quad d\pi dw \\
&= \frac{1}{\bar{q}}\left[Pr\left(w > T \ \& \ \pi w > \frac{\bar{q}}{p-1}\right) + BPr\left(w \leq T \ \& \ \pi w > \frac{\bar{q}}{B(p-1)}\right)\right] \\
&= \frac{1}{\bar{q}}\left[\int_T^1 \int_{\frac{\bar{q}}{w(p-1)}}^1 d\pi dw + B\int_{\frac{\bar{q}}{B(p-1)}}^T \int_{\frac{\bar{q}}{wB(p-1)}}^1 d\pi dw\right] \\
\therefore \mathcal{S} &= \frac{1}{\bar{q}}\left[1 + T(B-1) - \frac{\bar{q}}{p-1} + \frac{\bar{q}}{p-1}\log\left(\frac{\bar{q}}{B(p-1)}\right)\right].
\end{aligned}
$$

Implicit function theorem gives us:

$$
\left.\frac{\partial \bar{q}}{\partial B}\right|_{B=1} = \bar{q}_0 \frac{T - \frac{\bar{q}_0}{p-1}}{1 - \frac{\bar{q}_0}{p-1}} > 0.
$$

The human capital is given by:

$$
H = \int_0^1 \int_0^1 \left(1 - Pr^{in}\right)\pi + Pr^{in} p\pi \ d\pi dw = \frac{1}{2} + (p-1)\int_0^1 \int_0^1 Pr^{in}\pi \ d\pi dw,
$$

where

$$\int_0^1 \int_0^1 Pr^{in}\pi \, d\pi dw = \frac{1}{\bar{q}} \left[\int_T^1 \int_{\frac{\bar{q}}{w(p-1)}}^1 \pi \, d\pi dw + B \int_{\frac{\bar{q}}{B(p-1)}}^T \int_{\frac{\bar{q}}{wB(p-1)}}^1 \pi \, d\pi dw \right]$$
$$= \frac{1}{2} \left[\frac{1 - T + BT}{\bar{q}} + \frac{\bar{q}}{(p-1)^2} - \frac{\bar{q}}{(p-1)^2 T} - \frac{2}{p-1} + \frac{\bar{q}}{B(p-1)^2 T} \right].$$

Finally, the marginal effect of the policy is given by:

$$\left. \frac{\partial H}{\partial B} \right|_{B=1} = (p-1) \left[\frac{\partial}{\partial B} \int_0^1 \int_0^1 Pr^{in}\pi \, d\pi dw \Big|_{B=1} + \frac{\partial}{\partial \bar{q}} \int_0^1 \int_0^1 Pr^{in}\pi \, d\pi dw \Big|_{B=1} \frac{\partial \bar{q}}{\partial B}\Big|_{B=1} \right]$$
$$= \frac{1}{2T} \left(T - \frac{\bar{q}_0}{p-1} \right) (1 - T) > 0.$$

Given that $T \in \left(\frac{\bar{q}_0}{p-1}, 1 \right)$, affirmative action has a positive effect on aggregate human capital.

D Numerical Solution

The numerical solution of our model is similar to that of Restuccia and Urrutia (2004). We use a Markov approximation for the innate ability process, as suggested by Tauchen (1986), and define a grid over the state space. For a given grade point cutoff and for a given government expenditure on early education, we solve the value and policy functions numerically. We adjust the government expenditure on early education and the grade point cutoff until the government budget is balanced, and the admissions market is cleared. We solve the model doing the following steps:

(i) Guess the value function for the old parents, government expenditure on early education and grade point cutoff;

(ii) Solve for the young parent's value and policy functions, taking as given the old parent's value function;

(iii) Solve for the value and policy functions of the old parent, taking the young parent's value function as given;

(iv) Iterate (ii) and (iii) until convergence;

(v) Guess a distribution over the state space for young parents, and, using the step (iv) young parent policy function, compute the old parent state space distribution. Using the implied distribution and the old parent policy function, compute the young parent state space distribution. Repeat until convergence.

(vi) Lastly, set the government expenditure on early education and the grade point cutoff to balance the government budget and clear the admissions market. Using these new values, repeat the previous steps until convergence.

In order to improve the numerical precision of general equilibrium conditions, we use taste shocks as described in Gordon (2019) in all discrete choices in the model. We pick a taste shock variance parameter of 0.0001. We verify that taste shocks do not alter the quantitative meaning of the computations by certifying that the numbers in the calibration table do not change significantly if we turn taste shocks off: the largest absolute change in moment values happens for the college wage premium which falls from 2.66 with taste shocks to 2.64 without taste shocks.

We compute the efficiency-maximizing affirmative action policy by choosing the bonus points of each income quintile that maximizes aggregate output. The welfare-maximizing policy achieves the highest average welfare. We construct an initial educated guess for the numerical optimization by running a grid search varying bonus points by 10 p.p. from 0% to 100%. In the numerical optimization, policy bonus points are a multiple of 5 p.p. in order to prevent the procedure from becoming computationally intense.

E Moments' selection following Daruich (2018)

We follow Daruich's methodology closely by analyzing all calibrated parameters at the same time.[37] We first define a nine-dimensional hypercube for all nine calibrated parameters. Then, we draw 3,000 parameter vectors from a uniform Sobol quasi-random point set. Each draw consists of a 9-tuple for all parameters. For each of the randomly-generated parameterization, we solve the model and compute the two model-implied moments associated with each respective parameter according to Table 3.

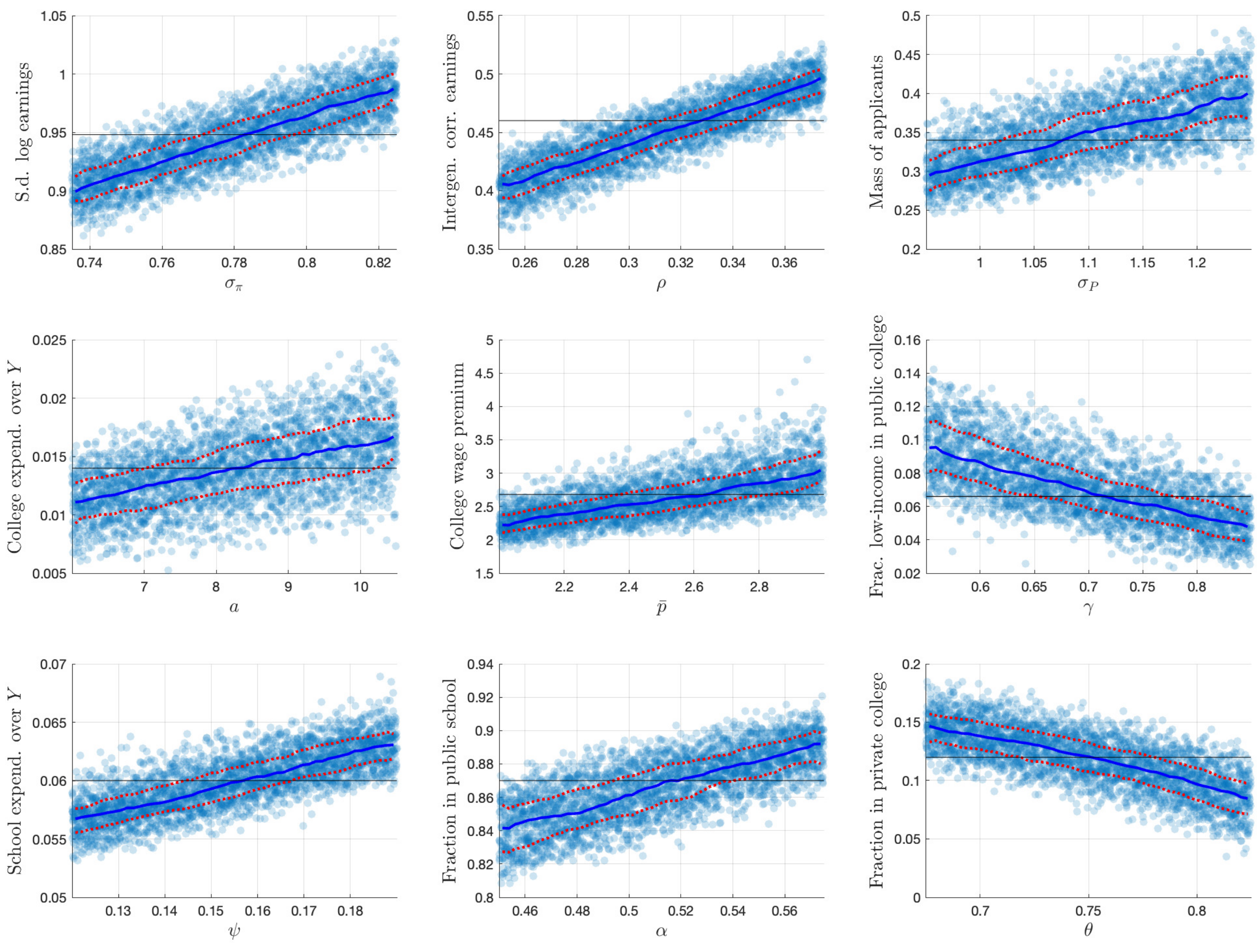
Finally, in a rolling window for each parameter, we compute the quartiles of the model-implied moment associated with this parameter across all the simulated parameterizations in that window. In each panel of Figure E.1, we plot the extreme quartiles as dashed red lines and the median as a solid blue line. In addition, we plot blue dots representing each of the simulated points from the hypercube and we also plot a straight line representing with the moment observed in the data (solid black line). Each panel reports a moment from Table 3 in the y-axis and its respective parameter in the x-axis.

According to Daruich (2018), for a well-identified model, we should expected that each model-implied moment to change steeply as we vary the parameter it is associated with, even tough the other parameters are being randomly chosen from the Sobol set. This is exactly

[37] See Appendix C.4 in Daruich (2018).

Figure E.1: Identification

This figure reports sensitivity of model-implied moments with respect to parameters of the model, following the methodology developed by Daruich (2018). We first define a nine-dimensional hypercube for all nine calibrated parameters. Then, we draw 3,000 parameter vectors from a uniform Sobol quasi-random point set. Each draw consists of a 9-tuple for all parameters. For each of the randomly-generated parameterization, we solve the model and compute the two model-implied moments associated with each respective parameter according to Table 3. In a rolling window for each parameter, we compute quartiles of the model-implied moment associated with this parameter across all the simulated parameterizations in that window. In each panel, we plot the extreme quartiles as dashed red lines and the median as a solid blue line. In addition, we plot blue dots representing each of the simulated points from the hypercube and we also plot a straight line representing with the moment observed in the data (solid black line). Each panel reports a moment from Table 3 in the y-axis and its respective parameter in the x-axis.



what we find. The figure shows that all model-implied moments change monotonically with respect to each respective parameter. The means that we vary each parameter on the x-axis its respective model-implied moments changes monotonically even though all other parameter are randomly chosen in the hypercube.

Moreover, the magnitude of the inter-quartile range indicates how important of the other paramaters are for each specific moment. In each panel of Figure E.1, the inter-quartile ranges is narrow enough that both quartiles are away from the value observed in the data (solid black line) at the extreme values of each parameter. This indicates that each moment is sensitive to a distinct parameter in the model but not as sensitive to other parameters. For example, in the first panel we have that standard deviation of log earnings is very sensitive to the volatility innate ability innovations (σ_π) because the blue line is steep, but not as sensitive to other parameters as the inter-quartile ranges (the distance between the red dashed lines) are narrow and even away from the moment observed in the data for extreme values of σ_π. Overall, we have narrow inter-quartile ranges with monotonic quartiles in all panels, suggesting that each model-implied moment is sensitive to a distinct parameter in the model.

F Transition Dynamics

The policy evaluation discussed in Section 5 results from comparing two steady state equilibria: one with no policy in place and another with affirmative action in college admissions. The comparison between steady states allows us to infer long-term implications of the policy, however it abstracts away from the transition between steady states. In this section, we numerically solve for the transition dynamics. To this end, we start off from a no-affirmative-action steady state equilibrium and then the policy is exogenously implemented at period T, after which the economy evolves towards the new steady state with an affirmative action policy in place.

Formally, we solve for the path of equilibrium prices—i.e. the grade point cutoffs, government expenditure in early education, and wage rates—such that, in each transition period, the government has a balanced budget and the labor markets as well as the market of college admissions clear. At each period during the transition, agents know the entire transition path and optimally adjust their consumption and investment decisions accordingly. We also assume that the convergence to the new steady state takes at most 15 periods after the intervention and the new steady state is achieved in the 16^{th} period after the policy was first implemented. This is a generous non-binding 240-year cap on the transition dynamics.

We analyze the transition dynamics of the efficiency-maximizing policy discussed in Section 5.2.[38] In Figure F.2, we report the evolution of fraction of students admitted to public

[38]The transition in the welfare-maximizing policy is similar.

college from different income quintiles; aggregate output; and social welfare. First, it is clear that the policy successfully changes college demographics quickly. In contrast, the convergence of output and welfare is more gradual. In Figure F.2, we also report the evolution of the equilibrium wages in the skilled and unskilled labor markets.

As soon as it is implemented, the policy changes college demographics by admitting more low-income applicants with higher returns to college education and fewer high-income applicants with lower returns to college education. On the one hand, when the policy is implemented, it benefits the most the lower-income-quintiles families, leading to a large initial welfare gain for two reasons: (i) these families have higher marginal utilities; and, (ii) there is a significant increase in the skilled aggregate human capital (see the second row in Figure F.2). On the other hand, upon implementation of the policy, there is a decline in the human capital of the non-college educated workers leading to a decrease in unskilled aggregate human capital. Paired with the changes in the wages, this leads to a decrease in aggregate output immediately after the policy implementation. As new generations arrive, families adjust their investment decisions; the applicants that are being displaced by the policy change their educational investments and the unskilled aggregate human capital starts growing; the economy converges to the new steady state with higher aggregate output and higher welfare.

Figure F.2: Transition Dynamics

This figure reports the transition dynamics from the steady state equilibrium without affirmative action to the steady state equilibrium with the efficiency-maximizing policy in place. We assume that the policy is exogenously implemented at period T, after which the economy starts to evolve towards the new steady state equilibrium with an affirmative action policy in place. In the first row, we report the change in the fraction of student from different income quintiles; in the second row, we report the output gain and the welfare gain; in the third row, we report the evolution in the aggregate unskilled and skilled human capitals. Finally, the last row reports the changes in wages for the skilled and unskilled labor markets.